

# Counter-Credit-Risk Yield Spreads: A Puzzle in China's Corporate Bond Market

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## Abstract

In this paper, using China's risk-free and corporate zero yields together with aggregate credit risk measures and various control variables from 2006 to 2013, we document a puzzle of counter-credit-risk corporate yield spreads. We interpret this puzzle as a symptom of the immaturity of China's credit bond market, which reveals a distorted pricing mechanism latent in the fundamental of this market. We also find interesting results about relationships between corporate yield spreads and interest rates as well as risk premia and the stock index, and these results are somewhat attributed to this puzzle. [JEL: G12, E43]

**Keywords:** China's Corporate Bond Market; Credit Risk; Term Structure of Yields

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# 1 Introduction

In China, banks have long been the primary source of capital. Total outstanding bank loans is ten times larger than that of credit bonds (Ma, 2006; Suzuki et al., 2008).<sup>1</sup> Chinese government itself has realized that this highly skewed financial system accumulates systemic risks. Regulatory reform, with officials clearing away some of the obstacles that have stood in the way of the development of the bond market (Zhou, 2003, 2006; Mu, 2006), has been constantly advancing the development of China's credit bond market. After the People's Bank of China (PBC) launched Commercial Papers (CPs) in 2005, the China's credit bond market entered into its fast growth period. By 2012, the annual issuance of the whole market is RMB 2 trillion that is about seven times the amount in 2005. With this speed of growth, the credit bond market is becoming more and more important in China. Given China's increasingly important and influential role in the global economy, this market will soon become one of the indispensable members in the international investment community. Therefore, it deserves timely and careful studies that target not only Chinese but broader research audience. Hale (2007) and Chen et al. (2011) describe the current status and the recent developments of China's credit bond market. Huang and Zhu (2009) provide an overview of the history of China's credit bond market, although their focus is on the whole bond market. To the best of our knowledge, however, there has been no previous literature that provides quantitative empirical studies on this market.

Using a set of comprehensive zero yield curve data of China's government bonds and credit bonds, along with China's aggregate credit risk measures, macroeconomic variables, and an aggregate liquidity measure from 2006 to 2013, we surprisingly find a puzzle of counter-credit-risk corporate yield spreads. In other words, we find a significant and negative relationship between the corporate yield spreads and aggregate credit risk measures. This result is completely counterintuitive given the fact that a corporate bond market is naturally expected to discover the price of credit risks in a directly proportional way.

Specifically, we first use a three-factor Gaussian term structure model to capture the

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<sup>1</sup> "Credit bond" is a term unique to the Chinese market. It is similar to corporate bond in the general sense. However, "Corporate bond" is used in China to represent a sub-market of the whole "Credit bond" market. The relationship between different terms will become clear as we move on to the section describing China's credit bond market.

dynamic of the term structure of risk-free interest rates. Then we develop a five-factor affine defaultable bond model, in which the first three factors coming from the interest rate model represent the risk factors of the interest rates, and the other two factors (CIR processes) capture the corporate-specific component (the component in the corporate yield spread that is uncorrelated to the risk-free interest rate factors) implicit in the term structure of corporate yield spreads. This five-factor model is fitted to the corporate zero yield curve data of bonds with different ratings. Results of the model allow us to study a) the relationship between corporate yield spreads and risk-free interest rates; b) the dynamics of the corporate-specific component; and c) the dynamics of corporate-specific risk premia. We find that the corporate yield spreads are negatively related to the level of the risk-free interest rates, but positively related to the slope and curvature of the interest rates. The positive relationships with the slope and curvature, which contradict theoretical predictions (Longstaff and Schwartz, 1995; Collin-Dufresne et al., 2001), might be attributed to the counter-credit-risk puzzle.

We then regress the model implied corporate-specific components on the aggregate credit risk measures. The regression results formally confirm the counter-credit-risk puzzle: the corporate-specific components are significantly and negatively related to the aggregate credit risk measures. The results are robust to different aggregate credit risk measures and to the inclusion of various control variables. The control variables include: the non-performing bank loan rate, the stock index, the GDP growth rate, and an aggregate bond market liquidity measure. Next, we regress the model implied corporate-specific risk premia on the aggregate credit risk measures and the control variables. We find that the risk premia are not consistently and significantly related to the credit risk, and they are, however, significantly and negatively related to the GDP growth. This is consistent with the evidence in Adrian et al. (2010) about the relationship between the GDP growth and the macro risk premium. For higher (lower) ratings they have a significantly positive (negative) relationship with the stock index (the non-performing bank loan rate). This might indicate that for the credit bond investors the equity and bank loan markets are comparable substitutes in the sense that when the equity market and/or bank loan market perform better (worse) the opportunity cost of investing in the credit bond market becomes higher

(lower), therefore the requested risk premium is higher (lower). The results of the risk premia help us understand the cause and mechanism behind the puzzle.

We interpret this puzzle as one of the symptoms of the immaturity of China's credit bond market. The liquidity in the secondary market is very low, which prevents the bond prices from revealing the true underlying risks. At the same time, zero default experience in the past two decades plus the explicit or implicit guarantees provided by high profile parent companies or local governments attract credit risk sensitive capital when the overall credit condition deteriorates. This distorted pricing mechanism might funnel the credit risks to the credit bond market instead of diversifying the credit risks. The emergence of the symptom alerts policy makers to focus more on the secondary market development and correction of the credit risk pricing mechanism.

The remainder of this paper is organized as follows. Section 2 provides a brief introduction to China's credit bond market. Section 3 describes the data, and presents results of some preliminary analyses, which already reveal the counter-credit-risk puzzle. In Section 4 we derive the term structure models that are used to identify the corporate-specific risk factors and risk premia from risk-free and corporate zero yields. Section 5 presents formal empirical analyses. Section 6 concludes the paper. Appendices contain technical details and supplemental empirical results.

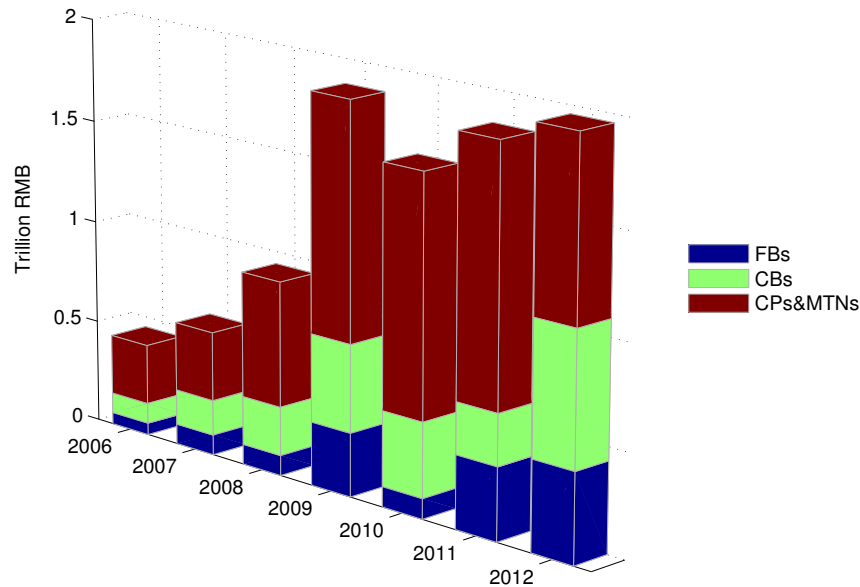
## **2 China's credit bond market**

China's credit bond market started in 1983. While before 2005 the development of the market was rather slow, it has been rapidly developing since 2005 in which the CPs were launched by the PBC in the interbank market. In 2008, the National Association of Financial Market Institutional Investors (NAFMII) launched Medium Term Notes (MTNs) which further advanced the development of the market.

China's credit bond market consists of three major submarkets: the financial corporate bond market (FB), the corporate bond market (CB), and the commercial papers and medium term notes market (CPs&MTNs). The FBs are mainly issued by commercial banks, the CBs and the CPs&MTNs are issued by non-financial firms. Most of the CBs

**Figure 1:** Annual bond issuance by the three major credit bond markets

Source: [www.ChinaBond.com.cn](http://www.ChinaBond.com.cn).



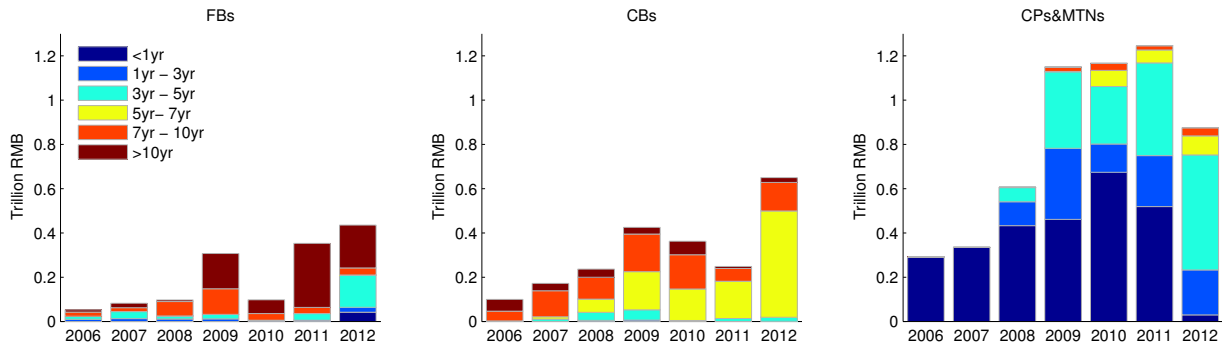
have explicit guarantees, while the FBs and the CPs&MTNs have no explicit guarantees. It is commonly believed, however, that all the credit bonds are implicitly guaranteed by local governments. Especially, given the crucial role of banks in China, the fact that the FBs are mostly issued by commercial banks makes the FBs almost equivalent to the government bonds.

Figure 1 shows annual issuances in these three markets from 2006 to 2012. The development of the markets first peaked in 2009, where the total issuance was about RMB 1.9 trillion. It then decreased to RMB 1.6 trillion in 2010, and peaked in 2012 again with a total issuance close to RMB 2 trillion. By the end of 2012, the CPs&MTNs captured 45 percent of the market share, the largest share held by any of the three markets. The CB accounted for 33 percent, and the FB held 22 percent of the market share. The FB, the CPs&MTNs, and a majority of the CB are based in the interbank market. Only a small portion of the CB is based in the exchange market. In total, 95 percent of the credit bond trading is conducted in the interbank market.

The three markets cover different parts of the term structure. As shown in Figure 2, the FB mainly issues bonds with maturities longer than five-year; the CB is mainly for bonds

**Figure 2:** Annual bond issuance by different maturities

Source: [www.ChinaBond.com.cn](http://www.ChinaBond.com.cn).



with maturities ranging from three to ten-year, and the CPs&MTNs mainly issues bonds with maturities shorter than five-year. In other words, the FB, CB, and CPs&MTNs cover the long end, the middle part, and the short end of the term structure, respectively.

Unlike the corporate bond market in the U.S., where the Financial Industry Regulatory Authority (FINRA) is the only regulator that oversees the entire market, four major regulating agencies oversee the Chinese market: the PBC, the NAFMII, the National Development Reform Commission (NDRC), and the China Securities Regulatory Commission (CSRC). The PBC and the NAFMII regulate the FBs and the CPs&MTNs, respectively. The NDRC oversees most of the CBs<sup>2</sup>, and the CSRC only regulates a small portion of the CBs that are issued by listed firms and traded in the exchange market.

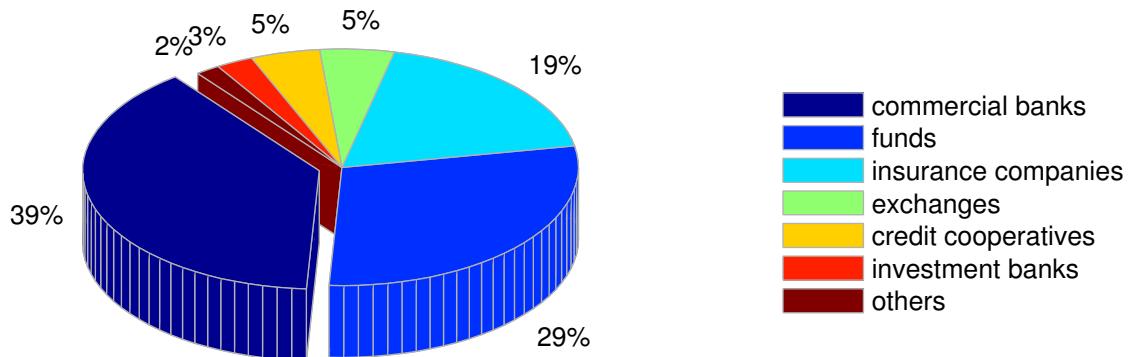
Figure 3 presents the holding constitution in the whole credit bond market by the end of 2012. We can see that commercial banks, funds, and insurance companies are the three largest bond holders in the market, and they hold 39, 29, and 19 percent of all the bonds, respectively. They are followed by exchanges, credit cooperatives, and investment banks who account for five, five, and three percent of the holdings, respectively.

Despite the rapid growth of the credit bond market, pressing issues need to be addressed. The secondary market remains weak, where the majority of players are financial institutions commonly engaged in “buy and hold” trading strategies. Thus the liquidity in the market is very low. Default happens from time to time in a mature corporate bond market, however, there had not been any default in China’s credit bond market until 21

<sup>2</sup> In China this majority is named “Enterprise Bonds”.

**Figure 3:** Bond holding constitution in China's credit bond market

Source: [www.ChinaBond.com.cn](http://www.ChinaBond.com.cn). The constitution is calculated based on data up to the end of 2012.



years after it started.<sup>3</sup> The local governments likely helped avert default, and the government had been showing “zero tolerance” for default until recently. For a corporate bond market, proper bond defaults are beneficial instead of toxic. A bond default would aid the development of risk pricing models by providing a precedent for how much asset value investors could expect to recover in the event of default (Wildau, 2012). These issues might be the cause of the counter-credit-risk corporate yield spreads puzzle documented in this paper.

### 3 Data, preliminary analyses and results

#### 3.1 Risk-free and corporate zero yields data

As previously mentioned that the FBs are issued by financial institutions, mostly commercial banks. In China, big banks are deemed to be of little credit risk. The bonds issued by some of the big banks, such as the China Development Bank and the Export-Import Bank of China, are commonly regarded as risk-free bonds. A big portion of FB is made of bonds

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<sup>3</sup> Shandong Helon was a recent case of technical default, however, it was averted by an implicit bailout from the Weifang local government. <http://ftalphaville.ft.com/2012/04/09/952211/avoiding-a-first-ever-corporate-default-in-china/>; On 7 Mar 2014, Chaori Solar Energy Science & Technology became the first ever Chinese company to default on its corporate bonds, after the government refrained from bailing it out. <http://www.telegraph.co.uk/finance/china-business/10682394/China-allows-first-ever-corporate-bond-default.html>

like these. Thus the FB, which only accounts for a market share of about 20 percent, is not representative enough of China’s credit bond market. Given this fact, we focus on the CB and the CPs&MTNs. Results of the CB are presented in the main text, and those of the CPs&MTNs are presented in the online appendix. The zero yield data used in the paper are from China Central Depository & Clearing Co. Ltd (CCDC) that is believed to be the leading authority on providing bond yield curve products in China.<sup>4</sup> The CB has seven ratings: AAA, AA+, AA, AA-, A+, A, and BBB.<sup>5</sup> For each rating and the risk-free interest rates, we use the same term structure: one-, two-, three-, five-, seven-, and ten-year. Data frequency is daily.

Table 1 reports the summary statistics of the data as well as the starting and ending dates of each rating. One thing worth noting is that, as shown in the last column of Table 1, lower ratings not only come with higher yield spreads but also higher volatilities. Figure 4 presents the time series dynamics of the zero yields and yield spreads. In the past 8 years, there were two peaks in the dynamics of the yield spreads. The first peak happened around late 2009, and the second occurred early 2011.

### 3.2 Left hand side and right hand side variables

In this subsection, we try to obtain some preliminary sense on how the credit yield spreads are related to various factors. Basically, we consider four types of variables: aggregate credit risk, business cycle, equity market performance, and aggregate bond market liquidity. For aggregate credit risk, the first variable we consider is the Value-Weighted Corporate Vulnerability Index ( $CVI_{vw}$ ) of China produced by the Risk Management Institute (RMI) at the National University of Singapore (NUS). This index provides a bottom-up (based on probabilities of default (PDs) of all individual listed firms) measure of the aggregate credit risk in China. The PDs are calculated based on historical equity market defaults and financial statements data of listed firms.<sup>6</sup> Of course the “default” events

<sup>4</sup> The yield curves are constructed from observed transaction bond prices using Hermite interpolation. Details are available at [http://eyield.chinabond.com.cn/cbweb/doc/doc\\_en.htm](http://eyield.chinabond.com.cn/cbweb/doc/doc_en.htm).

<sup>5</sup> We take all ratings available to us. Unfortunately, they do not include A-, nor do they break down BBB into the finer notches.

<sup>6</sup> Briefly speaking, the PDs are outputs of a parametric function, which links the PDs to Macro economic variables and firm-specific variables. The parameters of this function are frequently calibrated using the most updated data such that the resulting PDs match the historical default rates as well as possible.



**Table 1:** Summary statistics of risk-free interest rates and corporate yield spreads

This table reports summary statistics of the risk-free zero yields and corporate yield spreads at 1yr, 5yr, and 10yr of the seven ratings (AAA, AA+, AA, AA-, A+, A, and BBB). The corporate yield spread is defined as the difference between corporate bond zero yield and risk-free zero yield. Starting dates of the data are in the second column, all data end 1-Apr-2013. Median, Mean, Maximum (Max), Minimum (Min), and Standard Deviation (STD) of each category are reported. All figures are in percentages.

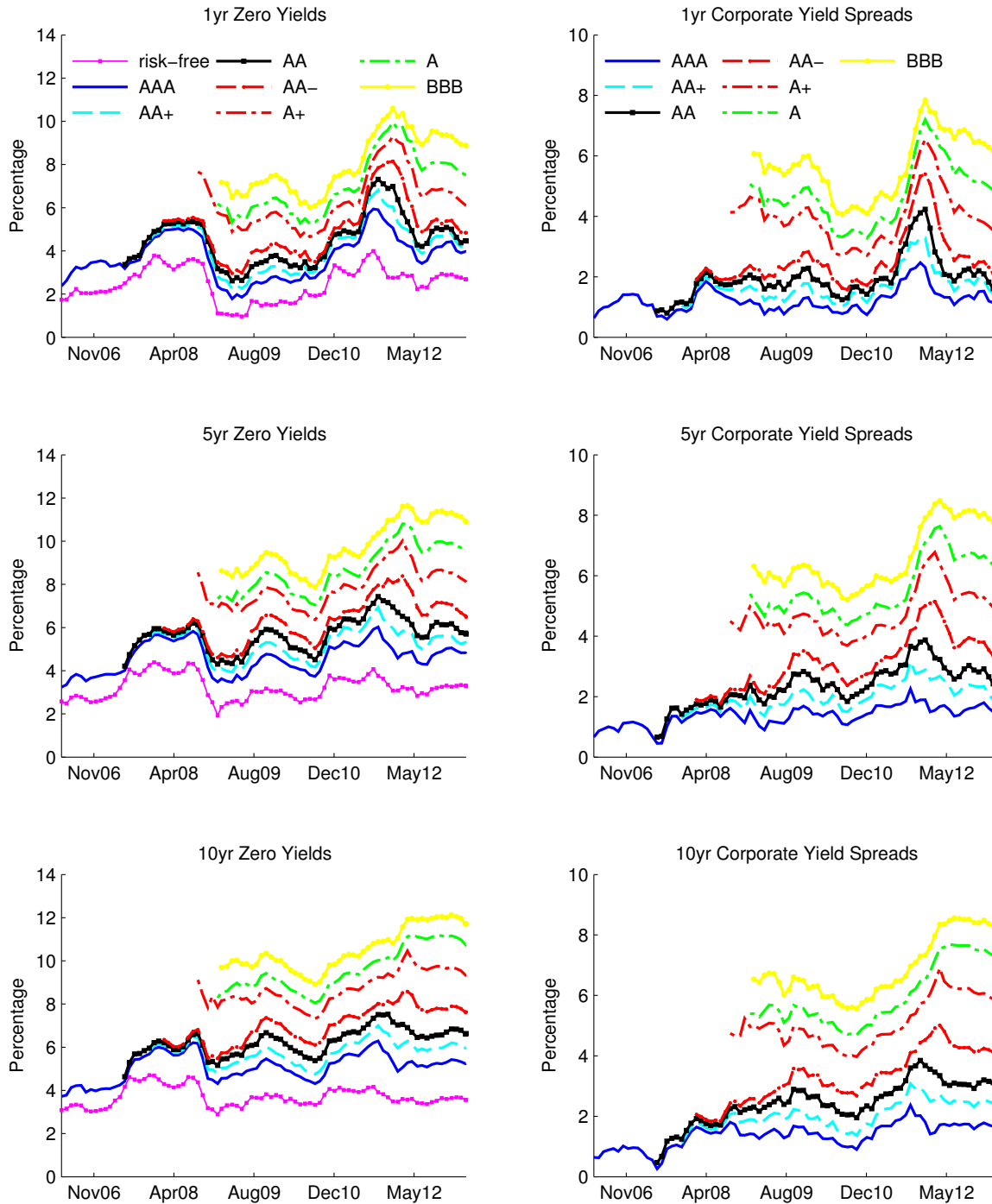
	Start from	Maturity	Median	Mean	Max	Min	STD
risk-free	01-Mar-2006	1 yr	2.474	2.456	4.020	0.920	0.775
		5 yr	3.150	3.221	4.489	1.845	0.571
		10 yr	3.619	3.705	4.768	2.823	0.463
AAA	01-Mar-2006	1 yr	1.138	1.193	2.684	0.317	0.385
		5 yr	1.402	1.341	2.384	0.281	0.343
		10 yr	1.420	1.342	2.461	0.166	0.411
AA+	11-Oct-2007	1 yr	1.585	1.679	3.621	0.839	0.500
		5 yr	1.938	2.001	3.315	1.212	0.406
		10 yr	1.997	2.048	3.178	1.092	0.437
AA	25-May-2007	1 yr	1.841	1.939	4.258	0.713	0.675
		5 yr	2.333	2.364	3.873	0.501	0.648
		10 yr	2.412	2.451	3.899	0.375	0.693
AA-	07-Jan-2008	1 yr	2.325	2.536	5.467	1.514	0.857
		5 yr	3.074	3.115	5.149	1.680	0.864
		10 yr	3.244	3.287	5.136	1.739	0.854
A+	21-Aug-2008	1 yr	3.951	4.009	6.599	2.626	0.902
		5 yr	4.523	4.757	6.865	3.648	0.749
		10 yr	4.882	5.092	6.914	3.871	0.759
A	25-Dec-2008	1 yr	4.644	4.747	7.247	3.225	0.998
		5 yr	5.175	5.591	7.651	4.329	0.943
		10 yr	5.539	5.970	7.790	4.591	1.010
BBB	06-Jan-2009	1 yr	5.664	5.700	7.900	3.997	1.022
		5 yr	6.109	6.602	8.498	5.168	1.044
		10 yr	6.485	6.875	8.605	5.456	0.999

identified by RMI are not defaults on equity since, technically speaking, the owner of a firm has no obligation to the equity holders. So these defaults are essentially credit events related to problematic debts (e.g. defaults on bank loans) and bankruptcy. Based on RMI-Staff (2014),<sup>7</sup> the following three events: bankruptcy filing, a missed or delayed payment

<sup>7</sup> See details in Section II.4, and Table A.9. <http://d.rmicri.org/static/pdf/2014update1.pdf>

**Figure 4:** Zero yields and yield spreads

In this figure, the three subplots on the left, from top to bottom, show the dynamics of 1yr, 5yr, 10yr zero yields (the risk-free yield and corporate yields of the seven ratings: AAA, AA+, AA, AA-, A+, A, and BBB), respectively; the three subplots on the right, from top to bottom, show the dynamics of 1yr, 5yr, 10yr corporate yield over risk-free yield spreads of the seven ratings.



of interest and/or principal, and debt restructuring/distressed exchange are recognized to be defaults in China's equity market. In its database, RMI has collected noticeable amount of defaults in China's equity market via various sources (RMI-Staff, 2014). However, there

were no defaults in China's credit bond market until recently. More details about the CVI can be found in RMI-Staff (2012).

The second variable is the one year forward average default rate (1yr-fwd ave DR). This measure is constructed by taking the average of monthly default rates in the next 12 months.<sup>8</sup> The default rate is the ratio of the number of defaulted listed firms to the number of active listed firms. Specifically, the time  $t$  1yr-fwd ave DR is given by

$$\text{1yr-fwd ave DR}_t = \frac{1}{12} \sum_{s=t+\frac{1}{12}}^{t+1} \text{DR}_s$$

where  $\text{DR}_s = \frac{\text{\# of defaults at time } s}{\text{\# of actives at time } s}$ . These default data are from the global credit risk database maintained by RMI NUS. It is commonly accepted that default rates of an economy reveal the lagged information about the aggregate credit condition of this economy, and this measure is often used as a diagnostic of the reasonableness of a PD model (see, e.g., Duffie et al., 2007; Duan et al., 2012). Thus this measure is informative about the aggregate credit risk.

The third variable is the aggregate non-performing bank loan rate (NPLR) which is the ratio of non-performing loans to total loans. The data are collected from the website of China Banking Regulatory Commission (CBRC).<sup>9</sup> In general, NPLR can be used as a credit risk measure (Altman et al., 1998; Martinez Peria and Mody, 2004; Jappelli et al., 2005). This measure aggregates NPLR of big-, medium-, and small-sized banks, in which the latter two categories account for around half of the total bank loans in China. Most of the loan borrowers of medium- and small-sized banks are small- and medium-sized enterprises (SMEs) (see, e.g., Shen et al., 2009). Therefore, NPLR somewhat reflects the credit risk of the SMEs market, where the majority of players are smaller firms that have only bank loans and do not issue corporate bonds. We do acknowledge, however, that NPLR might not be a pure credit risk measure as it can also be affected by some non-credit factors, such as overall banking performance, banking regulatory policy.

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<sup>8</sup> The monthly 1yr-fwd ave DR data are fitted to weekly frequency by assigning the same figure to the weeks within a month.

<sup>9</sup> CBRC website: <http://www.cbrc.gov.cn/chinese/home/docViewPage/110009.html> (accessed 18-Feb-2014). The quarterly NPLR data are fitted to weekly frequency by assigning the same figure to the weeks within a quarter.

For equity market performance and business cycle, we use Shanghai Stock Exchange Composite Index (SSECI) and China's quarterly GDP growth (GDPG).<sup>10</sup> Since our analysis is based on aggregate data, i.e., the zero yields of corporate bonds for different ratings, using equity market index and macroeconomic variables is consistent with Huang and Kong (2003). They find that credit spread changes for high-yield bonds are more closely related to equity market factors and also provide evidence in favor of incorporating macroeconomic factors into credit risk models.

For aggregate bond market liquidity measure (LM), we use a modified measure of temporary price changes accompanying order flow proposed by Pastor and Stambaugh (2003). The frequency of the LM is weekly. The higher the LM, the more the liquidity. More details of the LM are outlined in the online appendix.

Among all these variables, we focus on aggregate credit risk measures. In three aggregated measures, the CVI provides the most accurate and updated information about the credit risk, as it comprehensively and efficiently combines historical physical defaults, financial statements, and equity market information; 1yr-fwd ave DR follows next to CVI, as it utilizes pure defaults information; NPLR is the least accurate, as it also measures the performance of the banking industry and is not a pure credit risk measure. Therefore NPLR along with SSECI, GDPG, and LM are treated as control variables.

In the regression analyses, we do not consider the time series differences of the variables, even though some of the variables are non-stationary, for the following two reasons: a) given the relatively short time span of the data and the low frequency of certain factor (quarterly GDPG and NPLR), by taking differences we will have to throw away massive portion of the zero yield data in order to match the quarterly frequency. This will essentially leave us less than 23 data points from which we can barely carry out any reliable regression analysis; b) when variables have a strong dynamic structure, i.e., the current value depends significantly on the lagged ones, differencing the variables does not ensure the unbiased estimates (see the online appendix for details). Therefore in this study, we focus on relationships between levels of factors. The left hand side (LHS) variable is the average Yield Spreads (YS) across the different maturities, the right hand side (RHS) vari-

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<sup>10</sup> The data of SSECI are from Wind, and the data of GDPG are from Zheng and Wang (2010). The quarterly GDPG data are fitted to weekly frequency by assigning the same figure to the weeks within a quarter.

ables include  $CVI_{vw}$ , 1yr-fwd ave DR, NPLR, GDPG SSECI, and LM. From Figure 4, we can see that the YS are persistent processes. Therefore, we also include two lagged YS on the RHS to remove the autocorrelation in regression residuals. We call this specification Lagged Dependent Variable (LDV) model. This is recommended firstly by Granger and Newbold (1974), and followed by many ever since (see Keele and Kelly, 2006; Han, 2008, etc.).

In order to have a clear presentation for coefficient estimates (not too large or too small), we keep the left hand side (LHS) variable in percentage and adjust the magnitude of the RHS variables as follows: we log  $CVI_{vw}$ , GDPG, and SSECI, then divide them by 10 (GDPG is in percentage, CVI is in bps); divide NPLR by 10 (NPLR is in percentage); keep 1yr-fwd ave DR (1yr-fwd ave DR is in percentage) and LM intact. As  $CVI_{vw}$  and 1yr-fwd ave DR are significantly related the control variables, we use adjusted log  $CVI_{vw}$  (adj-log CVI) and adjusted 1yr-fwd ave DR (adj-ADR), which are orthogonal to NPLR, log GDPG, log SSECI, and LM, so that the coefficients of CVI, 1yr-fwd ave DR, the other variables can clearly represent YS's correlations with the credit risk and the other risk measures, respectively.

### 3.3 Preliminary regression analyses

For each rating, we run the following regressions,

$$\begin{aligned} YS_t &= \text{Intercept} + \beta_1 \text{adj-log } CVI_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t \\ &\quad + \beta_4 \log \text{SSECI}_t + \beta_5 \text{LM}_t + \beta_6 YS_{t-2\Delta t} + \beta_7 YS_{t-\Delta t} + \epsilon_t; \\ YS_t &= \text{Intercept} + \beta_1 \text{adj-ADR}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t \\ &\quad + \beta_4 \log \text{SSECI}_t + \beta_5 \text{LM}_t + \beta_6 YS_{t-2\Delta t} + \beta_7 YS_{t-\Delta t} + \epsilon_t, \end{aligned}$$

where  $\Delta t$  is one week.<sup>11</sup>

One issue with the regressions is the non-stationarity of the YS processes. For most of the ratings, the Dickey-Fuller test fails to reject the unit root hypothesis of YS. In order to validate the LDV when LHS is non-stationary, we conduct extensive Monte Carlo sim-

<sup>11</sup> This is also the case in regressions in later empirical sections. However, the term structure models in the next section are estimated using daily data to reduce the discretization impact on MLE estimates.

**Table 2:** Results from the regression of YS on adj-log CVI and other control variables

In this table the results from the following regression

$$YS_t = \text{Intercept} + \beta_1 \text{adj-log CVI}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t + \beta_4 \log \text{SSECI}_t \\ + \beta_5 \text{LM}_t + \beta_6 \text{YS}_{t-2\Delta t} + \beta_7 \text{YS}_{t-\Delta t} + \epsilon_t,$$

are reported for all the seven ratings (AAA, AA+, AA, AA-, A+, A, BBB) as well as the average. The sample first order autocorrelation of the residuals  $\rho$  is also reported in the last column. In the table, \*\*\*, \*\*, and \* entries represent significance at the 1%, 5%, and 10% levels, respectively. The significance levels are calculated based on the Bootstrapped standard errors detailed in Appendix A.

	Intrcpt	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\beta_7$	$\rho$
AAA	<b>0.824</b> ***	<b>-5.307</b> ***	-0.001	-0.234	0.042	<b>-0.111</b> *	0.082	<b>0.873</b> ***	0.009
AA+	<b>1.618</b> ***	<b>-9.787</b> ***	-0.003	-0.227	-0.085	-0.132	0.068	<b>0.864</b> ***	0.001
AA	<b>1.306</b> ***	<b>-7.921</b> ***	-0.002	-0.273	-0.077	-0.109	0.014	<b>0.954</b> ***	-0.000
AA-	<b>1.325</b> ***	<b>-7.471</b> ***	-0.001	-0.490	-0.163	-0.121	0.017	<b>0.974</b> ***	0.020
A+	<b>3.321</b> ***	<b>-9.499</b> ***	<b>-0.026</b> ***	<b>-0.941</b> *	<b>-2.087</b> ***	-0.049	0.071	<b>0.910</b> ***	0.044
A	<b>2.078</b> ***	-1.745	<b>-0.120</b> ***	<b>-1.959</b> ***	<b>-1.534</b> ***	-0.068	-0.065	<b>1.063</b> ***	0.022
BBB	<b>2.323</b> ***	-3.721	<b>-0.114</b> ***	<b>-1.872</b> ***	<b>-1.507</b> **	-0.074	-0.080	<b>1.074</b> ***	0.017
average	<b>1.170</b> ***	<b>-6.497</b> **	-0.000	-0.011	-0.314	<b>-0.229</b> **	0.062	<b>0.939</b> ***	0.004

ulations in the online appendix and show the finite sample appropriateness of the OLS estimates of a linear model similar to the above LDV. The results are robust to the presence of the unit root processes (both LHS and RHS), series correlated residuals, endogeneity between LHS and RHS, and measurement errors in both LHS and RHS. Although the OLS estimates are shown to work well, the standard OLS or Newey-West standard errors are no longer valid in presence of non-stationarity. We therefore use the Bootstrapped standard errors, of which the computation is detailed in Appendix A, to derive the statistical inference of the coefficient estimates.

The regression results are reported in Tables 2 and 3. From these results the most prominent observation is that the YS are significantly and negatively related to the aggregate credit risk measures. In Table 2, for most of the ratings the coefficients of adj-log CVI are negative and significant (AAA, AA+, AA, AA-, and A+ are at 1% level, the average is at 5%

**Table 3:** Results from the regression of YS on adj-ADR and other control variables

In this table the results from the following regression

$$\begin{aligned} \text{YS}_t = & \text{Intercept} + \beta_1 \text{adj-ADR}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t + \beta_4 \log \text{SSECI}_t \\ & + \beta_5 \text{LM}_t + \beta_6 \text{YS}_{t-2\Delta t} + \beta_7 \text{YS}_{t-\Delta t} + \epsilon_t. \end{aligned}$$

are reported for all the seven ratings (AAA, AA+, AA, AA-, A+, A, BBB) as well as the average. The sample first order autocorrelation of the residuals  $\rho$  is also reported in the last column. In the table, \*\*\*, \*\*, and \* entries represent significance at the 1%, 5%, and 10% levels, respectively. The significance levels are calculated based on the Bootstrapped standard errors detailed in Appendix A.

	Intrcpt	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\beta_7$	$\rho$
AAA	<b>0.236*</b>	<b>-0.483***</b>	-0.002	-0.292	0.052	<b>-0.098*</b>	0.082	<b>0.848***</b>	0.017
AA+	0.236	<b>-0.481***</b>	-0.002	-0.189	-0.017	<b>-0.139*</b>	0.073	<b>0.881***</b>	0.014
AA	0.202	<b>-0.361***</b>	-0.001	-0.214	-0.040	<b>-0.116*</b>	0.007	<b>0.974***</b>	0.015
AA-	0.329	<b>-0.439***</b>	0.001	-0.470	-0.170	-0.125	0.025	<b>0.975***</b>	0.031
A+	<b>1.764***</b>	<b>-0.561***</b>	<b>-0.026***</b>	-0.718	<b>-1.832***</b>	-0.058	0.070	<b>0.924***</b>	0.043
A	<b>1.810***</b>	-0.147	<b>-0.120***</b>	<b>-1.922***</b>	<b>-1.503***</b>	-0.072	-0.059	<b>1.059***</b>	0.024
BBB	<b>1.723***</b>	-0.277	<b>-0.113***</b>	<b>-1.768***</b>	<b>-1.430***</b>	-0.085	-0.067	<b>1.068***</b>	0.020
average	<b>0.356*</b>	<b>-0.417***</b>	-0.000	-0.007	-0.312	<b>-0.228**</b>	0.058	<b>0.944***</b>	0.002

level). In Table 3, we observe the same significantly negative relationship between the YS and adj-ADR. For ratings A+, A, and BBB, the YS are significantly and negatively related to NPLR. These results are strikingly counter-intuitive, as a corporate bond market is expected to positively price the credit risk by design. The significantly negative relationship between YS of more highly rated bonds and credit risks is undoubtedly puzzling. This result seems to be consistent with a speculation that there is a flight-to-safety effect that is at work, since the effect is stronger for more highly rated bonds, among all rated ones.

For the business cycle variable, we find that only the YS of the lower rated bonds (A+, A, and BBB in Table 2; A and BBB in Table 3) are significantly related to the log GDPG and the relationship is negative. Therefore, we only observe moderate counter-cyclical pattern if we just look at the model-free YS. Similarly, for the equity market performance measure, log SSECI, only coefficients of lower ratings are significantly negative. For the liquidity

measure, LM, although we do not find consistently significant results across ratings, we do have some significantly negative coefficients for higher ratings and the average rating.

In this preliminary regression analysis, although we do already see a puzzling counter-credit-risk pattern in the corporate bond market by just looking at the YS, the analysis offers no more insights about the cause and mechanism of this puzzle. In particular, we are not able to say anything about the risk premia in this market which is the key to understanding this puzzle. For the purpose of better understanding this puzzle, in the following sections we develop pricing models for risk-free and defaultable bonds, from which we derive the model implied YS and corporate-specific risk premia. In the formal empirical analyses, we further analyze this puzzle with the help of the risk premium measures.

## **4 Term structure models for risk-free and defaultable Bonds**

The modeling specification we adopt is an extended and modified version of the one developed by Duffee (1999). In the original specification, the risk-free zero coupon bond is modeled using a two-factor CIR model, and the credit spread is modeled using a one-factor CIR model. First, we extend the former and the latter to three-factor and two-factor models, respectively. This extension allows us to have greater flexibility in fitting the term structure of both risk-free interest rates and credit spreads. Second, we use a three-factor Gaussian Dynamic Term Structure Model (GDTSM) instead of the original multi-factor CIR model for the risk-free interest rates, while keeping the CIR specification for modeling the credit spread. This modification is motivated by the factor that multi-factor CIR models are typically less flexible in fitting risk-free term structure than the GDTSMs with the same number of factors (see, e.g., Dai and Singleton, 2002), while they perform reasonably well in fitting credit spreads (see, e.g., Duffee, 1999; Longstaff et al., 2005; Ang and Longstaff, 2013). Also, in light of recent development of the GDTSMs (Joslin et al., 2011), a GDTSM is much easier to estimate, and can be directly linked to principal components (PCs) of the term structure of the risk-free interest rates. Making use of the PCs is helpful for interpreting the coefficients. Filipović and Trolle (2013) also adopt a “mixed” specification (Gaussian setup for risk-free factors, and CIR for credit spread factors) in their term



structure modeling.

#### 4.1 Risk-free zero coupon bond

Here we use a variant of the classic GDTSM,  $\mathbb{A}_0(3)$ , for risk-free zero-coupon bond. The state variables are specified to follow a dynamic process as

$$dZ_t = \mathbf{A}Z_t dt + \mathbf{B}dW_t,$$

under the measure  $\mathbb{Q}$ , where

$$\mathbf{A} = \begin{bmatrix} -k_1 & 0 & 0 \\ 0 & -k_2 & 0 \\ 0 & 0 & -k_3 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \Omega_1 & 0 & 0 \\ \Omega_2 & \Omega_4 & 0 \\ \Omega_3 & \Omega_5 & \Omega_6 \end{bmatrix}.$$

Then the risk-free zero-coupon bond is given by

$$P(t, T) = \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^T r_s ds \right) \right] = \exp \left( \mathbf{H}(T - t) - \mathbf{F}(T - t)^\top Z_t \right)$$

where

$$\begin{aligned}\mathbf{H}(x) &= -\left(\varphi - \frac{1}{2}\mathbf{C}\left(\mathbf{A}^{-1}\mathbf{B}\mathbf{B}^\top(\mathbf{A}^\top)^{-1}\right)\mathbf{C}^\top\right)x - \int_0^x \mathbf{C}(s)\left(\mathbf{A}^{-1}\mathbf{B}\mathbf{B}^\top(\mathbf{A}^\top)^{-1}\right)\left(\mathbf{C}^\top - \frac{\mathbf{C}(s)^\top}{2}\right)ds, \\ \mathbf{F}(x)^\top &= \mathbf{C} \int_0^x \exp(\mathbf{A}s) ds = (\mathbf{C}(x) - \mathbf{C})\mathbf{A}^{-1}, \\ \mathbf{C}(x) &= \mathbf{C} \exp(\mathbf{A}x) \\ \mathbf{C} &= [1, 1, 1].\end{aligned}$$

Appendix B contains detailed derivation of the model.

## 4.2 Defaultable zero coupon bond

Using the results from Duffie and Singleton (1997, 1999), a defaultable zero-coupon bond is given by

$$B(t, T) = \mathbb{E}_t^Q \left[ \exp \left( - \int_t^T (r_s + h_s) ds \right) \right],$$

where  $h_t$  is the spread between adjusted discount rate and risk-free discount rate and can consist of a liquidity intensity process, and the product of a default intensity process and a fractional default loss process (the fraction of *market value* of the bond that is lost upon default). It might seem tempting to model the liquidity process separately as done in Driessen (2005). However, Driessen (2005)'s liquidity process has to be anchored to an observable liquidity proxy in order to estimate its parameters. We do not want to anchor our liquidity process to the LM, given the fact that it does not show a consistently significant relationship with the YS (see Tables 2 and 3). Therefore, we do not model the liquidity process separately at this stage to retain the modeling simplification, and consider it in the later regressions analysis after  $h_t$  being estimated from the data. Similarly, we do not model the randomness of the loss given default. The (almost) zero default experience in China's credit bond market makes it somewhat lame to expect any reasonable uncertainty in the recovery rates.

As we use the model to price the term structure of corporate yields of different ratings,  $h_t$  can be understood as the average spread of bonds of the rating. Notice that the first two principal components of the panel data of YS for each rating explain over 96% of the

overall variation of the term structure. For simplicity, we assume the callability/puttability of the corporate bonds is mostly related to the risk free interest rate and captured in  $h_t$  by a linear relation with the interest rate state variables. Therefore, we model  $h_t$  to be an affine function of the interest rate state variables and two corporate-specific factors:

$$h_t = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & 1 & 1 \end{bmatrix} \begin{bmatrix} PC_t \\ \lambda_{1,t} \\ \lambda_{2,t} \end{bmatrix},$$

where  $\lambda_{i,t}$  under the measure  $\mathbb{Q}$  follows:

$$d\lambda_{i,t} = \kappa_i (\theta_i - \lambda_{i,t}) dt + \sigma_i \sqrt{\lambda_{i,t}} dw_{\lambda_{i,t}},$$

$i = 1, 2$ .  $w_{\lambda_{i,t}}$  and  $W_t$  are mutually independent.  $PC_t$  contains the first three principal components of the risk-free zero curve, and follows

$$dPC_t = \mathbf{A}^{PC,\mathbb{Q}} (PC_t - \mu^{PC,\mathbb{Q}}) dt + \mathbf{B}^{PC} dW_t$$

under the measure  $\mathbb{Q}$ , where  $\mathbf{A}^{PC,\mathbb{Q}}$ ,  $\mu^{PC,\mathbb{Q}}$ , and  $\mathbf{B}^{PC}$  are given in Appendix B. To be concise, we rewrite the state variables as the vector

$$Y_t = \underbrace{[PC_t^\top, \lambda_{1,t}, \lambda_{2,t}]^\top}_{5 \times 1},$$

and its dynamic, in terms of notations in Duffie et al. (2000), under the measure  $\mathbb{Q}$  is given by

$$dY_t = (K_0 + K_1 Y_t) dt + \Sigma(Y_t) dW_{Y,t}$$

where

$$\begin{aligned}
K_0 &= \begin{bmatrix} -\mathbf{A}^{PC,Q} \mu^{PC,Q} \\ \kappa_1 \theta_1 \\ \kappa_2 \theta_2 \end{bmatrix}, K_1 = \begin{bmatrix} \mathbf{A}^{PC,Q} & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \begin{bmatrix} -\kappa_1 & 0 \\ 0 & -\kappa_2 \end{bmatrix} \end{bmatrix} \\
\Sigma(Y_t) \Sigma(Y_t)^\top &= \begin{bmatrix} \mathbf{B}^{PC} (\mathbf{B}^{PC})^\top & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \end{bmatrix} \odot \text{diag}(Y_t) \\
W_{Y,t} &= \underbrace{[W_t^\top, w_{\lambda_1,t}, w_{\lambda_2,t}]^\top}_{5 \times 1}.
\end{aligned}$$

$\odot$  denotes the element-wise multiplication,  $\text{diag}(\bullet)$  expands the vector  $\bullet$  into a diagonal matrix in which the diagonal is  $\bullet$ .

Given these notations,

$$\begin{aligned}
B(t, T) &= \mathbb{E}_t^{\mathbb{Q}} \left[ \exp \left( - \int_t^T \left( \varphi^{PC} + \begin{bmatrix} \delta_1 + \rho_1, & \delta_2 + \rho_2, & \delta_3 + \rho_3, & 1, & 1 \end{bmatrix} Y_s \right) ds \right) \right] \\
&= \exp(\alpha(T-t) - \beta(T-t)^\top Y_t),
\end{aligned}$$

where  $\varphi^{PC}$  and  $\rho_i, i = 1, 2, 3$  are given in Appendix B;  $\alpha(x)$  and  $\beta(x)$  satisfy the ODEs

$$\begin{aligned} \frac{d\alpha(x)}{dx} &= -K_0^\top \beta(x) + \frac{1}{2} \beta(x)^\top \begin{bmatrix} \left( \mathbf{B}^{PC} (\mathbf{B}^{PC})^\top \right)_{3 \times 3} & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} \end{bmatrix} \beta(x) - (\delta_0 + \varphi) \\ \frac{d\beta(x)}{dx} &= K_1^\top \beta(x) - \frac{1}{2} \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sigma_1^2 \beta_4(x)^2 \\ \sigma_2^2 \beta_5(x)^2 \end{bmatrix} + \begin{bmatrix} \delta_1 + \rho_1 \\ \delta_2 + \rho_2 \\ \delta_3 + \rho_3 \\ 1 \\ 1 \end{bmatrix}, \end{aligned}$$

with the initial conditions  $\alpha(0) = 0$  and  $\beta(0) = \mathbf{0}_{5 \times 1}$ ,

$$\begin{aligned} \beta_{1:3}(x) &= \begin{bmatrix} \beta_1(x) \\ \beta_2(x) \\ \beta_3(x) \end{bmatrix} = \left( \left( \mathbf{A}^{PC, Q} \right)^\top \right)^{-1} \left( \exp \left( \left( \mathbf{A}^{PC, Q} \right)^\top x \right) - I_{3 \times 3} \right) \begin{bmatrix} \delta_1 + \rho_1 \\ \delta_2 + \rho_2 \\ \delta_3 + \rho_3 \end{bmatrix}, \\ \beta_{3+i}(x) &= \left( 1 - \frac{2}{\phi_i e^{\xi_i x} + 1} \right) \frac{\xi_i}{\sigma_i^2} - \frac{\kappa_i}{\sigma_i^2}, \phi_i = \frac{\xi_i + \kappa_i}{\xi_i - \kappa_i}, \xi_i = \sqrt{\kappa_i^2 + 2\sigma_i^2}, i = 1, 2 \\ \alpha(x) &= - \int_0^x \left( K_0^\top \beta(s) - \frac{1}{2} \beta_{1:3}(s)^\top \mathbf{B}^{PC} \left( \mathbf{B}^{PC} \right)^\top \beta_{1:3}(s) \right) ds - \varphi x \\ &= \left( \mathbf{A}^{PC, Q} \mu^{PC, Q} \right)^\top \int_0^x \beta_{1:3}(s) ds + \sum_{i=1}^2 \frac{\kappa_i \theta_i}{\sigma_i^2} \left[ (\xi_i + \kappa_i) x - 2 \log \left( \frac{\phi_i e^{\xi_i x} + 1}{\phi_i + 1} \right) \right] \\ &\quad + \frac{1}{2} \int_0^x \beta_{1:3}(s)^\top \mathbf{B}^{PC} \left( \mathbf{B}^{PC} \right)^\top \beta_{1:3}(s) ds - \varphi x. \end{aligned}$$

### 4.3 Market prices of risks (MPR)

We use the essentially affine MPR (Duffee, 2002) for  $PC_t$ , and the extended affine MPR (Cheridito et al., 2007) for  $\lambda_{1,t}$ , and  $\lambda_{2,t}$ . Then the measure  $\mathbb{P}$  dynamic of  $Y_t$  is given by

$$dY_t = \left( K_0^{\mathbb{P}} + K_1^{\mathbb{P}} Y_t \right) dt + \Sigma(Y_t) dW_{Y,t}^{\mathbb{P}}$$

where

$$K_0^{\mathbb{P}} = \begin{bmatrix} -\mathbf{A}^{PC,\mathbb{P}} \mu^{PC,\mathbb{P}} \\ \kappa_1 \theta_1 + \eta_1^0 \\ \kappa_2 \theta_2 + \eta_2^0 \end{bmatrix}$$

$$K_1^{\mathbb{P}} = \begin{bmatrix} \mathbf{A}^{PC,\mathbb{P}} & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \begin{bmatrix} -(\kappa_1 + \eta_1^1) & 0 \\ 0 & -(\kappa_2 + \eta_2^1) \end{bmatrix} \end{bmatrix}.$$

### 4.4 Central tendency and risk premium

Here we use two CIR processes to capture the variation in  $h_t$  that is unrelated to the interest rate factors, i.e.,  $PC_t$ . To extract useful economic information, we apply an invariant transformation to the two CIR factors and interpret the resulting representation as a central tendency system (see, e.g., Jegadeesh and Pennacchi, 1996). This invariant transformation is done using both the  $\mathbb{Q}$  parameters and  $\mathbb{P}$  parameters.

**Using  $\mathbb{Q}$  Parameters:** Without loss of generality, we assume  $\kappa_1 > \kappa_2$ . Then the new system

$[\lambda_t, \psi_t^Q]^\top$  is defined by the invariant transformation as

$$\begin{bmatrix} \lambda_t \\ \psi_t^Q \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{\kappa_1 - \kappa_2}{\kappa_1} & 0 \end{bmatrix} \begin{bmatrix} \lambda_{2,t} \\ \lambda_{1,t} \end{bmatrix} + \begin{bmatrix} 0 \\ \theta_1 + \theta_2 - \frac{\theta_2}{\kappa_1} (\kappa_1 - \kappa_2) \end{bmatrix},$$

which yields

$$\begin{aligned} \begin{bmatrix} d\lambda_t \\ d\psi_t^Q \end{bmatrix} &= \begin{bmatrix} \kappa_1 & -\kappa_1 \\ 0 & \kappa_2 \end{bmatrix} \left( \begin{bmatrix} \theta_1 + \theta_2 \\ \theta_1 + \theta_2 \end{bmatrix} - \begin{bmatrix} \lambda_t \\ \psi_t^Q \end{bmatrix} \right) dt \\ &+ \begin{bmatrix} \sigma_2 & -\sigma_1 \\ \frac{\kappa_1 - \kappa_2}{\kappa_1} \sigma_2 & 0 \end{bmatrix} \text{diag} \left( \begin{bmatrix} \sqrt{\frac{\kappa_1}{\kappa_1 - \kappa_2} \psi_t^Q - \frac{\kappa_1 \theta_1 + \kappa_2 \theta_2}{\kappa_1 - \kappa_2}} \\ \sqrt{\lambda_t - \frac{\kappa_1}{\kappa_1 - \kappa_2} \psi_t^Q + \frac{\kappa_1 \theta_1 + \kappa_2 \theta_2}{\kappa_1 - \kappa_2}} \end{bmatrix} \right) \begin{bmatrix} dw_{\lambda,t} \\ dw_{\psi,t} \end{bmatrix}. \end{aligned}$$

**Using  $\mathbb{P}$  Parameters:** Without loss of generality, we assume  $\kappa_1 + \eta_1^1 > \kappa_2 + \eta_2^1$ . Then the new system  $[\lambda_t, \psi_t^P]^\top$  is defined by the invariant transformation as

$$\begin{bmatrix} \lambda_t \\ \psi_t^P \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1)}{\kappa_1 + \eta_1^1} & 0 \end{bmatrix} \begin{bmatrix} \lambda_{2,t} \\ \lambda_{1,t} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\kappa_1 \theta_1 + \eta_1^0}{\kappa_1 + \eta_1^1} + \frac{\kappa_2 \theta_2 + \eta_2^0}{\kappa_2 + \eta_2^1} - \frac{(\kappa_2 \theta_2 + \eta_2^0)(\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1))}{(\kappa_2 + \eta_2^1)(\kappa_1 + \eta_1^1)} \end{bmatrix},$$

it follows that

$$\begin{aligned}
\begin{bmatrix} d\lambda_t \\ d\psi_t^{\mathbb{P}} \end{bmatrix} &= \begin{bmatrix} \kappa_1 + \eta_1^1 & -(\kappa_1 + \eta_1^1) \\ 0 & \kappa_2 + \eta_2^1 \end{bmatrix} \left( \begin{bmatrix} \frac{\kappa_1\theta_1 + \eta_1^0}{\kappa_1 + \eta_1^1} + \frac{\kappa_2\theta_2 + \eta_2^0}{\kappa_2 + \eta_2^1} \\ \frac{\kappa_1\theta_1 + \eta_1^0}{\kappa_1 + \eta_1^1} + \frac{\kappa_2\theta_2 + \eta_2^0}{\kappa_2 + \eta_2^1} \end{bmatrix} - \begin{bmatrix} \lambda_t \\ \psi_t^{\mathbb{P}} \end{bmatrix} \right) dt \\
&+ \begin{bmatrix} & \sigma_2 & -\sigma_1 \\ \frac{\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1)}{\kappa_1 + \eta_1^1} \sigma_2 & 0 \end{bmatrix} \text{diag} \left( \begin{bmatrix} \sqrt{\frac{\kappa_1 + \eta_1^1}{\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1)}} \psi_t^{\mathbb{P}} - \frac{\kappa_1\theta_1 + \eta_1^0 + \kappa_2\theta_2 + \eta_2^0}{\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1)} \\ \sqrt{\lambda_t - \frac{\kappa_1 + \eta_1^1}{\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1)} \psi_t^{\mathbb{P}} + \frac{\kappa_1\theta_1 + \eta_1^0 + \kappa_2\theta_2 + \eta_2^0}{\kappa_1 + \eta_1^1 - (\kappa_2 + \eta_2^1)}} \end{bmatrix} \right) \begin{bmatrix} dw_{\lambda,t}^{\mathbb{P}} \\ dw_{\psi,t}^{\mathbb{P}} \end{bmatrix}.
\end{aligned}$$

Given these representations, we have

$$h_t = \begin{bmatrix} & & & \\ \delta_1 & \delta_2 & \delta_3 & 1 \end{bmatrix} \begin{bmatrix} PC_t \\ \lambda_t \end{bmatrix}.$$

Therefore the component in  $h_t$  that is unrelated to the interest rate factors is now captured by  $\lambda_t$ , and  $\psi_t^{\mathbb{Q}}$  and  $\psi_t^{\mathbb{P}}$  represent  $\lambda_t$ 's conditional central tendencies under the measure  $\mathbb{Q}$  and  $\mathbb{P}$ , respectively. As mentioned in Shiller (1979), a conditional central tendency can be used to represent public expectations. Therefore  $\psi_t^{\mathbb{Q}}$  and  $\psi_t^{\mathbb{P}}$  represent the public expectations under the measure  $\mathbb{Q}$  and the measure  $\mathbb{P}$ , respectively, and the difference between  $\psi_t^{\mathbb{Q}}$  and  $\psi_t^{\mathbb{P}}$ ,  $\psi_t^{\mathbb{Q}} - \psi_t^{\mathbb{P}}$ , is a measure of risk premium associated with corporate-specific risks that are unrelated to the interest rate risks. It is worth noting that the risk premium derived here should not be confused with the “default event risk premium” (see, e.g., Driessen, 2005; Jarrow et al., 2005). Here we do not model default event risk premium, as we adopt the conventional assumption that the default intensities are of the same values under both measures  $\mathbb{Q}$  and  $\mathbb{P}$ . This conventional assumption is empirically common (see Longstaff et al., 2005; Pan and Singleton, 2008; Longstaff et al., 2011, and many others) and theoretically sound (see Bai et al., 2012). What we are measuring here is the so called “diffusion” risk premium associated with the unpredictable variation in  $\lambda_t$ .



## 5 Empirical analyses

### 5.1 Estimation and fitting performance

The model for risk-free zero coupon bond is estimated using Joslin et al. (2011)'s approach. The technical details are included in the online appendix. Given the results of the risk-free bond model, the model for defaultable zero coupon bond is estimated for each rating by keeping the interest rate related parameters and states at their estimates from the risk free bond model estimation. The estimation method for the defaultable bond model is QMLE in conjunction with Kalman filter. In the Kalman filter, we approximate the transition density of the CIR process by a Gaussian density which matches the first two moments of the transition density. Details of this method can be found in Duan and Simonato (1999).

Parameter estimates for the risk-free bond model and the defaultable bond model are presented in Table 4 and Table 5, respectively. We can see most of the measure  $\mathbb{Q}$  parameters are statistically significant (the only exception is  $\delta_2$  for the rating AA), and a few measure  $\mathbb{P}$  parameters are insignificant (see the last two columns in Table 4 and Table 5). This is consistent with the tradition of term structure model estimation. The filtered intensities ( $h_t$ ) are presented along with the short rate in Figure 6. On average, bonds with lower ratings have higher intensities. In addition, the intensities are negatively related to the short rate, especially at the beginnings of 2008, 2009, 2010, and 2011 where the short rate and the intensities went in exactly opposite directions. This is also somewhat confirmed in the second column in Table 5 where significantly negative estimates of  $\delta_1$ , which measures the sensitivity of  $h_t$  to the level of interest rates, are presented. The next section discusses this in more detail.

We use two CIR processes to model the term structure of the yield spreads. This is similar to the two-factor CIR model for term structure of interest rates in Duffee (1999). Therefore, the two original (before transformation) CIR factors represent the level and slope of the term structure of the yield spreads. We order the two factor such that the first (second) factor is closely related to the slope (level) which has a strong (weak) tendency to mean-revert. From Table 5, we can see that the risk premia parameters for the slope factor are usually not significant, while the opposite is true for the level factor. This might

indicate that in China the risk of level shifting in the yield spread is better priced than that of slope changing.

The coefficients  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$  capture the sensitivity of  $h_t$  to  $PC_1$ ,  $PC_2$ , and  $PC_3$ , respectively. Using the U.S. data, Litterman and Scheinkman (1991) find that the first three principal components of the yield curves well represent the level, slope, and curvature, respectively. This is also true in China's yield curve data, and is confirmed numerically in Table 7, and graphically in Figure 7. Therefore, we interpret the coefficients  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$  as the sensitivity of  $h_t$  to the level, the slope, and the curvature, respectively.

From Table 5, we can see for all the ratings,  $\delta_1$  is significantly negative. This finding is empirically consistent with the evidence in Longstaff and Schwartz (1995), Duffee (1998), and Collin-Dufresne et al. (2001). In contrast, we find that  $\delta_2$  is significant and positive for most of the ratings, i.e.,  $h_t$  is significantly positively related to the slope of the risk-free yield curve which is an indication of expectations of future short rates. Two exceptions are in AA and AA- where  $\delta_2$  is positive but insignificant in AA and significantly negative in AA-. Similarly, we also find that  $\delta_3$  is significantly positive for all the ratings. This can also be interpreted as a positive relationship between  $h_t$  and expectations of future short rates, per Giese (2008) the curvature might represent expectations of interest rates even better than the slope. This finding is the opposite of what is found in Duffee (1998) and Collin-Dufresne et al. (2001). These observations can be related to the counter-credit-risk puzzle. In the online appendix of the paper we provide a detailed conjecture explaining how these observations can be potentially attributed to the counter-credit-risk puzzle.

We now turn to the fitting performance of the models. Table 6 reports a summary of the model's pricing errors (the difference between the actual market yield and the model implied yield). Generally speaking, our models are effective at capturing the dynamics of the term structure of interest rates and corporate bond yields. The performance of fitting interest rates is a bit better than that of fitting corporate bond yields in terms of maximum (Max), minimum (Min), standard deviation (STD), mean absolute error (MAE), and average variance ratio (AVR) in the table. This is likely because the risk-free zero coupon bond model has three free factors, while the defaultable zero coupon bond model has only two

**Table 4:** Estimates of risk-free term structure model

This table reports the ML estimates and standard errors (Std. Err) of the measure  $\mathbb{Q}$  parameters of the risk-free zero coupon bond model, as well as the OLS estimates and standard errors (Std. Err) of the VAR parameters of  $PC$ s. The dynamic of  $PC$ s is given by a VAR process:  $PC_t = E + FPC_{t-\Delta t} + \varepsilon_t$ , where  $\Delta t$  is one day. The measure  $\mathbb{P}$  parameters  $A^{PC,\mathbb{P}}$  and  $\mu^{PC,\mathbb{P}}$  are explicit functions of  $E$  and  $F$ . These functions are presented in the online appendix.

Model Parameters	Estimates	Std. Err	VAR Coefficients of $PC$ s	Estimates	Std. Err
$k_1$	0.0602	(0.0038)	$E_1 \times 100$	-0.0454	(0.0176)
$k_2$	0.5442	(0.0078)	$E_2 \times 100$	0.009	(0.0116)
$k_3$	1.8914	(0.025)	$E_3 \times 100$	-0.02	(0.009)
$\Omega_1$	-0.0094	(0.0004)	$F_{[1,1]}$	0.9986	(0.001)
$\Omega_2$	0.0154	(0.001)	$F_{[2,1]}$	0.0214	(0.004)
$\Omega_3$	-0.006	(0.0014)	$F_{[3,1]}$	0.0112	(0.0126)
$\Omega_4$	-0.0162	(0.0006)	$F_{[1,2]}$	-0.0014	(0.0008)
$\Omega_5$	0.0256	(0.0012)	$F_{[2,2]}$	0.9938	(0.0026)
$\Omega_6$	0.0116	(0.0006)	$F_{[3,2]}$	-0.0298	(0.0082)
$\varphi$	0.0518	(0.0006)	$F_{[1,3]}$	0.0006	(0.0006)
			$F_{[2,3]}$	-0.0032	(0.002)
			$F_{[3,3]}$	0.964	(0.0064)

free factors.<sup>12</sup> We also plot the average model yields against the average actual yields in Figure 5. The averages are taken across all the maturities. The figure gives us a good sense of how well the models perform in capturing the dynamics. The model implied dynamics are so close to the actual dynamics that we can barely see the differences.

<sup>12</sup> Although the defaultable zero coupon bond model has five factors, the first three are the interest rate factor, and they are not free during the estimation of the defaultable bond model. Therefore only two out of the five factors essentially contribute to the fitting performance.

**Table 5:** Estimates of the parameters related to corporate zero yields

This table reports the ML estimates of the defaultable zero coupon bond model for the seven ratings (AAA, AA+, AA, AA-, A+, A, and BBB). The parameters are estimated by fixing the risk-free zero coupon bond model's parameters at their point estimates reported in Table 4. Standard errors (s.e.) are given in parentheses.

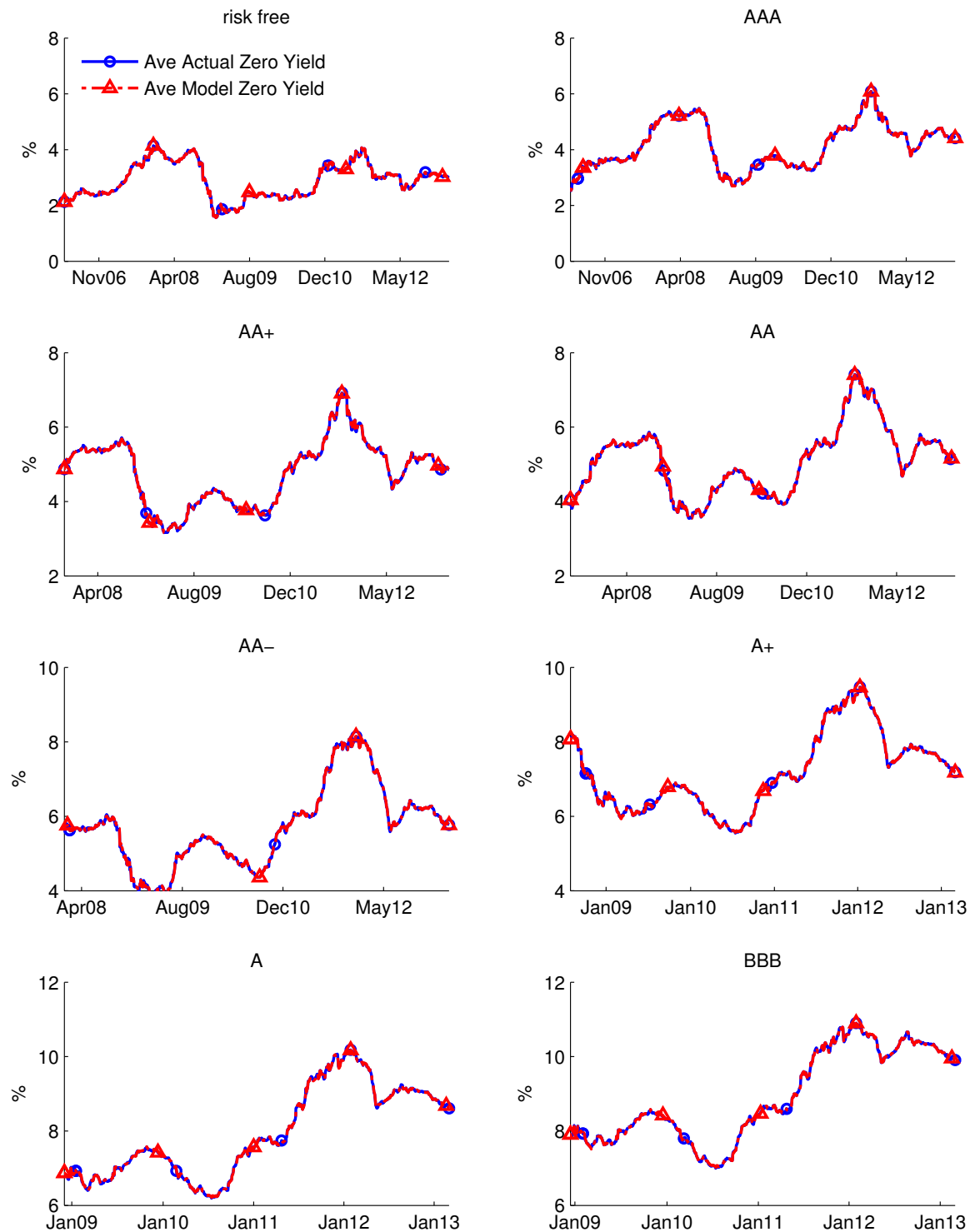
	$\delta_1$	$\delta_2$	$\delta_3$	$i$	$\kappa_i$	$\theta_i$	$\sigma_i$	$\eta_i^0$	$\eta_i^1$
AAA	-0.092	0.14	1.639	1	0.571	0.022	0.062	0.007	0.634
	(0.0106)	(0.021)	(0.0556)		(0.0168)	(0.001)	(0.0022)	(0.0044)	(0.3032)
				2	0.001	1.302	0.041	0.007	0.517
					(0.0002)	(0.3)	(0.0016)	(0.0032)	(0.1844)
AA+	-0.069	0.148	1.338	1	0.628	0.023	0.072	0.005	0.548
	(0.0172)	(0.0318)	(0.0668)		(0.0314)	(0.0016)	(0.0034)	(0.0052)	(0.3536)
				2	0.037	0.036	0.044	0.01	0.702
					(0.0116)	(0.0032)	(0.002)	(0.0044)	(0.2562)
AA	-0.145	0.041	1.438	1	0.518	0.032	0.069	0.004	0.302
	(0.0148)	(0.0452)	(0.071)		(0.0204)	(0.0012)	(0.002)	(0.0058)	(0.3106)
				2	0.035	0.055	0.04	0.038	1.381
					(0.0058)	(0.0044)	(0.0014)	(0.0212)	(0.9066)
AA-	-0.158	-0.12	1.515	1	0.537	0.046	0.065	-0.002	0.152
	(0.0172)	(0.0438)	(0.0942)		(0.022)	(0.0012)	(0.002)	(0.0074)	(0.2488)
				2	0.061	0.05	0.046	0.01	0.376
					(0.0048)	(0.0018)	(0.0014)	(0.0048)	(0.1372)
A+	-0.175	0.208	1.318	1	0.478	0.048	0.065	-0.008	-0.019
	(0.0182)	(0.0398)	(0.092)		(0.0284)	(0.0012)	(0.0024)	(0.0078)	(0.2328)
				2	0.076	0.061	0.047	0.014	0.495
					(0.011)	(0.0026)	(0.002)	(0.0064)	(0.2148)
A	-0.187	0.258	1.359	1	0.476	0.057	0.058	-0.015	-0.154
	(0.0152)	(0.0424)	(0.116)		(0.027)	(0.0016)	(0.0022)	(0.0082)	(0.2032)
				2	0.071	0.062	0.041	0.005	0.111
					(0.0082)	(0.0028)	(0.0016)	(0.0062)	(0.0564)
BBB	-0.168	0.084	1.607	1	0.592	0.065	0.06	-0.017	-0.171
	(0.0162)	(0.043)	(0.1302)		(0.024)	(0.0014)	(0.0022)	(0.0114)	(0.2266)
				2	0.125	0.057	0.047	0.004	0.063
					(0.0046)	(0.0022)	(0.0016)	(0.0054)	(0.0802)

## 5.2 Counter-credit-risk corporate yield spreads

Having identified the relationship between intensity  $h_t$  and interest rate factors, we are able to explore driving forces of the more corporate-specific factor  $\lambda$  which is the sum of

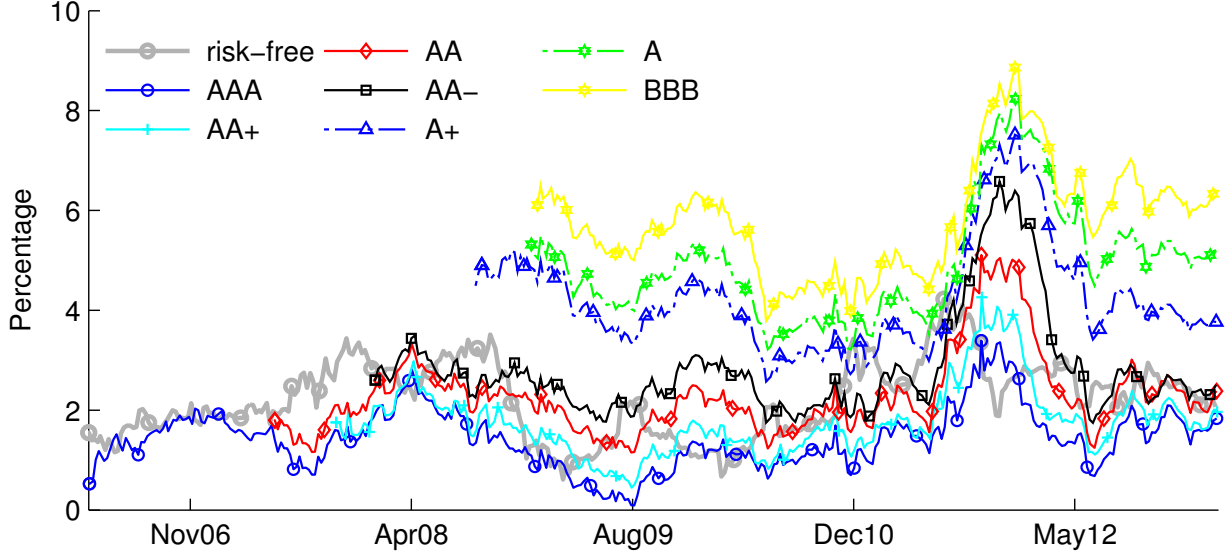
**Figure 5:** Average model yields vs average actual yields

The three subplots on the left, from top to bottom, show the dynamics of 1yr, 5yr, 10yr zero yields (the risk-free yield and corporate yields of the seven ratings: AAA, AA+, AA, AA-, A+, A, and BBB), respectively; while the three subplots on the right, from top to bottom, show the dynamics of 1yr, 5yr, 10yr corporate yield over risk-free yield spreads of the seven ratings.



**Figure 6:** Intensities ( $h_t$ ) and the risk-free short rate

In this figure, the dynamics of the intensity  $h_t = \sum_{i=1}^3 \delta_i PC_{i,t} + \sum_{i=1}^2 \lambda_{i,t}$  for all the seven ratings (AAA, AA+, AA, AA-, A+, A, and BBB) are plotted against the dynamic of the short rate  $r_t = \varphi^{PC} + \rho PC_t$ . The longest dynamic is from Apr 2006 to Mar 2013.



$\lambda_1$  and  $\lambda_2$  as derived in Section 4.4.

Tables 8 and 9 report the results from regressing  $\lambda$  of different ratings on credit risk and control variables as well as the lagged  $\lambda$ s, i.e., the results of the following regressions:

$$\begin{aligned} \lambda_t = & \text{Intercept} + \beta_1 \text{adj-log CVI}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t + \beta_4 \log \text{SSECI}_t \\ & + \beta_5 \text{LM}_t + \beta_6 \lambda_{t-2\Delta t} + \beta_7 \lambda_{t-\Delta t} + \epsilon_t, \end{aligned}$$

$$\begin{aligned} \lambda_t = & \text{Intercept} + \beta_1 \text{adj-ADR}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t + \beta_4 \log \text{SSECI}_t \\ & + \beta_5 \text{LM}_t + \beta_6 \lambda_{t-2\Delta t} + \beta_7 \lambda_{t-\Delta t} + \epsilon_t, \end{aligned}$$

where  $\Delta t$  is one week.

From the tables, we see almost the same results as Tables 2 and 3, except that we do not observe any significant relationship between  $\lambda$  and the LM. Specifically, we find that  $\lambda$  is significantly and negatively related to the aggregate credit risk measures. In contrast, only few of the coefficients of log SSECI and log GDPG are significant. Apparently the relationships between  $\lambda$  and business cycle and equity market performance are much weaker.

The negative relationship is also graphically shown in Figure 8. In Figure 8a to 8c, we

**Table 6:** Summary of the pricing errors

This table reports a summary of pricing errors of both the risk-free zero coupon bond model (fitted to the risk-free zero yields) and the defaultable zero coupon bond model (fitted to the corporate zero yields of the seven ratings AAA, AA+, AA, AA-, A+, A, and BBB). The pricing error is defined by the difference between the actual market yield and the model implied yield. Median, Mean, Maximum (Max), Minimum (Min), Standard Deviation (STD), Mean Absolute Error (MAE) of the pricing errors panel data (across all maturities and over time) for each category are reported. Average Variance Ratios (AVR) across all maturities for each category are also reported, where the variance ratio is defined as  $1 - \text{var}(\text{pricing error})/\text{var}(\text{yield})$ . Except AVR, all are in basis point.

	Median	Mean	Max	Min	STD	MAE	AVR
risk-free	0.019	0.004	21.615	-20.384	4.119	3.017	0.995
AAA	-0.079	0.001	34.514	-35.956	8.315	6.172	0.99
AA+	-0.005	-0.014	36.58	-36.034	9.613	7.277	0.985
AA	-0.17	-0.066	50.973	-34.77	10.395	8.024	0.986
AA-	0.291	-0.012	72.029	-49.036	11.351	8.425	0.99
A+	-0.229	-0.033	67.891	-58.847	12.685	9.303	0.984
A	-0.53	0.045	71.841	-59.547	13.847	10.107	0.987
BBB	-0.169	0.053	59.248	-47.697	13.234	10.019	0.989

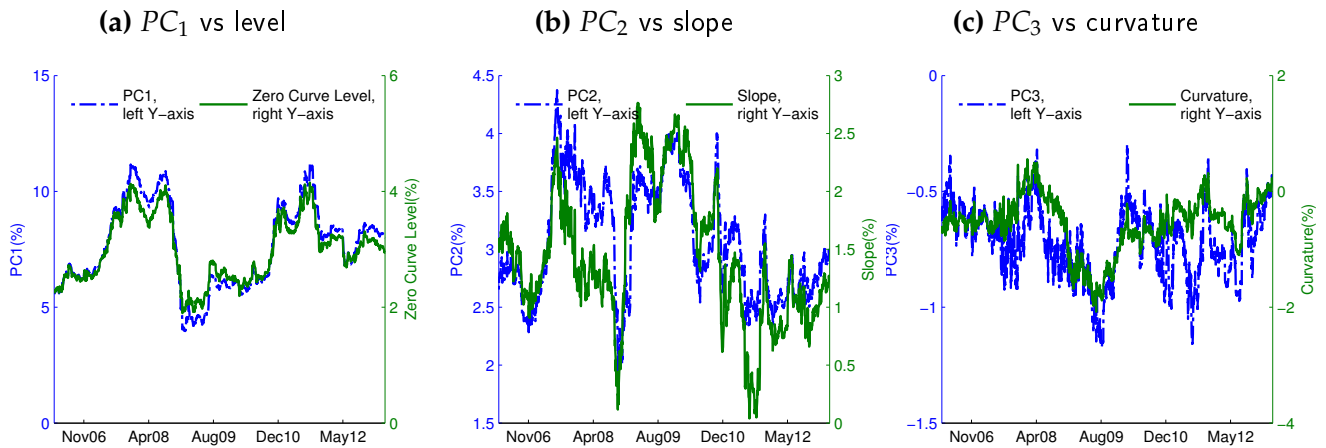
can clearly see the inverse relationship between levels of the average  $\lambda$  across the ratings and log CVI, 1yr-fwd ave DR, and NPLR. For log CVI, this level relationship is stronger in the sample after 2008, and for 1yr-fwd ave DR, it is all the way strong along the whole sample.

Based on the U.S. corporate bond data, Collin-Dufresne et al. (2001), Elton et al. (2001) among others show that corporate yield spreads are positively related to credit risks, even though the credit risk explains only a limited portion of corporate yield spreads.<sup>13</sup> We also find a positive correlation between Aaa, Baa corporate yield spreads and the U.S. CVI, and

<sup>13</sup> Huang and Huang (2012) use a calibration approach and show that credit risk accounts for only a small fraction of yield spreads for investment-grade bonds of all maturities. Based on individual corporate bonds, Eom et al. (2004) and Bao (2009) study the implication of structure models on the pricing of credit risks in corporate bonds. Collin-Dufresne et al. (2001) focus on changes in spreads and find that the unexplained portion is driven by factors that are independent of both credit-risk and standard liquidity measures. Elton et al. (2001) find that expected default losses account for a low fraction of spreads. They find that the Fama-French factors and state taxes can largely explain the credit risk premium.

**Figure 7:** PCs vs yield curve characteristics

The left panel of this figure plots the dynamics of  $PC_1$  and Level; the middle panel of this figure plots the dynamics of  $PC_2$  and Slope; the right panel of this figure plots the dynamics of  $PC_3$  and Curvature. Level is defined as the average of 3mth and 10yr zero yields; Slope is defined as the difference between 10yr and 3mth zero yields; Curvature is defined as  $2 \times (2\text{yr zero yield}) - (3\text{mth zero yield} + 10\text{yr zero yield})$ .



present the results in the online appendix. To the best of our knowledge, however, none of the previous studies has found any negative relationship between corporate yield spreads and credit risks. This negative relationship we document here undoubtedly presents a new puzzle in the area of empirical credit risk research. This puzzle is likely due to the immaturity of China's corporate bond market. The illiquid secondary market prevents the bond prices from revealing the true underlying risks. At the same time, the guarantees provided by high profile parent companies attract credit risk sensitive capital when the overall credit condition deteriorates. This distorted pricing mechanism might funnel the credit risks to the corporate bond market. Therefore, instead of diversifying the credit risks, the corporate bond market actually accumulates the credit risks, if the puzzle were a reflection of the above-mentioned distorted pricing mechanism. We discuss the cause of this puzzle in detail later. Before that we first take a look at the risk premium measure.



**Table 7:** Correlation between PCs and yield curve characteristics

The left panel of this table reports the correlation between PCs and level, slope, and curvature, the right panel of this table reports the correlation between first order differences of PCs and those of level, slope, and curvature. Level is defined as the average of 3mth and 10yr zero yields; Slope is defined as the difference between 10yr and 3mth zero yields; Curvature is defined as  $2*(2\text{yr zero yield}) - (3\text{mth zero yield} + 10\text{yr zero yield})$ .

Correlation in level			Correlation in first order difference		
$PC_1$ vs Level	$PC_2$ vs Slope	$PC_3$ vs Curvature	$PC_1$ vs Level	$PC_2$ vs Slope	$PC_3$ vs Curvature
0.991	0.764	0.655	0.915	0.901	0.964

**Table 8:** Results from the regression of  $\lambda$  on adj-log CVI and other control variables

In this table the results from the following regression

$$\lambda_t = \text{Intercept} + \beta_1 \text{adj-log CVI}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t + \beta_4 \log \text{SSECI}_t + \beta_5 \text{LM}_t + \beta_6 \lambda_{t-2\Delta t} + \beta_7 \lambda_{t-\Delta t} + \epsilon_t,$$

are reported for all the seven ratings (AAA, AA+, AA, AA-, A+, A, BBB) as well as the average. The sample first order autocorrelation of the residuals  $\varrho$  is also reported in the last column. In the table, \*\*\*, \*\*, and \* entries represent significance at the 1%, 5%, and 10% levels, respectively. The significance levels are calculated based on the Bootstrapped standard errors detailed in Appendix A.

	Intrcpt	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\beta_7$	$\varrho$
AAA	<b>1.82</b> ***	<b>-13.78</b> ***	-0.00	0.67	0.11	-0.04	0.03	<b>0.93</b> ***	-0.00
AA+	<b>2.03</b> ***	<b>-16.04</b> ***	-0.01	0.79	0.23	0.02	-0.05	<b>1.00</b> ***	-0.01
AA	<b>1.59</b> **	<b>-14.42</b> ***	-0.01	0.34	0.63	0.07	<b>-0.20</b> ***	<b>1.17</b> ***	0.02
AA-	<b>1.41</b> *	<b>-13.49</b> ***	-0.01	-0.09	0.79	0.10	-0.10	<b>1.08</b> ***	0.00
A+	<b>3.66</b> ***	<b>-16.11</b> ***	<b>-0.06</b> ***	-0.84	-1.28	0.02	-0.06	<b>1.04</b> ***	-0.01
A	<b>3.92</b> **	-9.82	<b>-0.21</b> ***	<b>-2.84</b> ***	-1.94	0.10	-0.07	<b>1.05</b> ***	0.00
BBB	<b>3.88</b> **	-10.92	<b>-0.23</b> ***	<b>-3.36</b> ***	-1.45	0.23	-0.04	<b>1.02</b> ***	-0.02
average	<b>1.22</b> **	<b>-8.43</b> **	-0.01	0.10	0.02	-0.04	<b>-0.18</b> ***	<b>1.16</b> ***	0.01

**Table 9:** Results from the regression of  $\lambda$  on adj-ADR and other control variables

In this table the results from the following regression

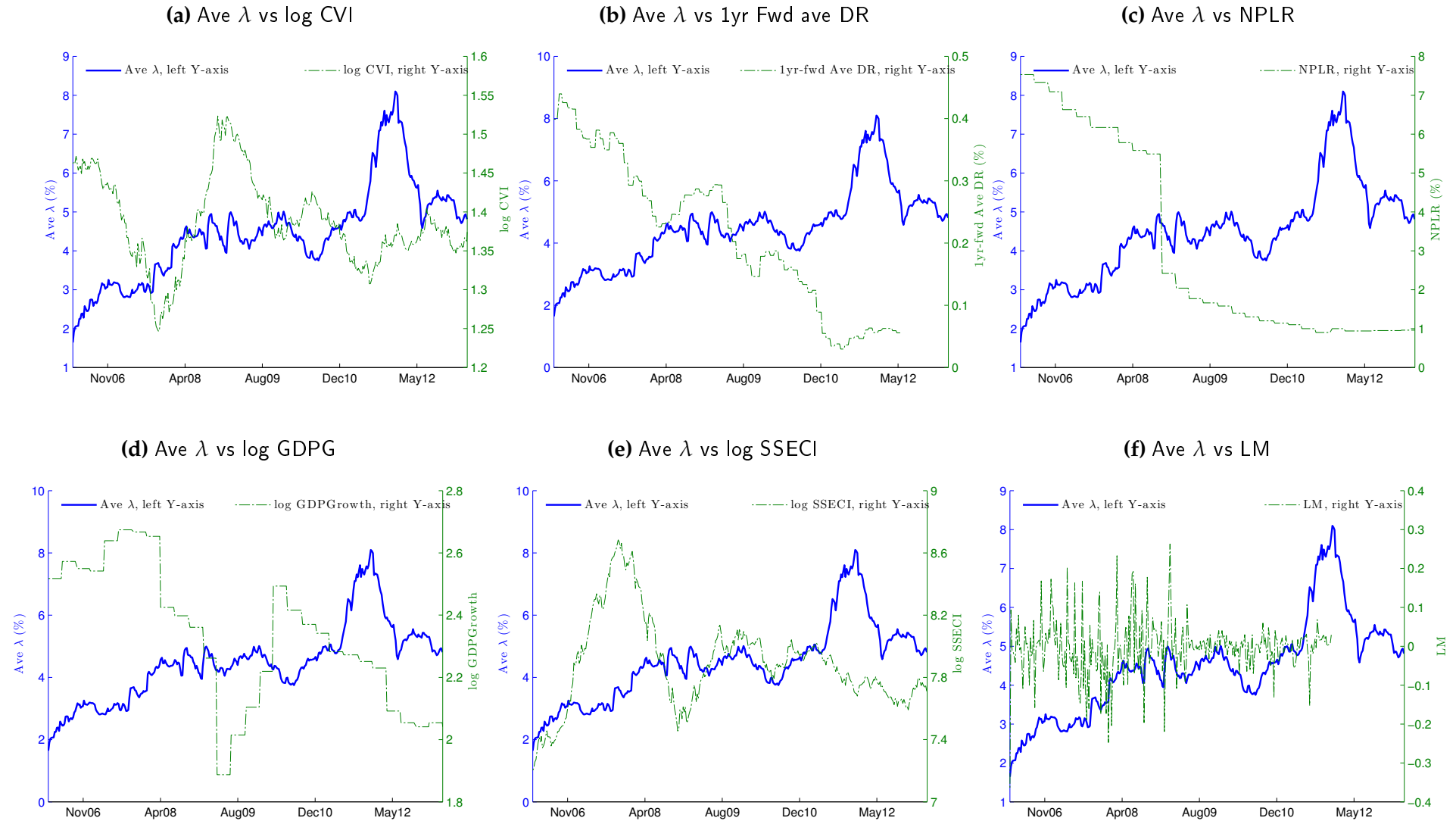
$$\lambda_t = \text{Intercept} + \beta_1 \text{adj-ADR}_t + \beta_2 \text{NPLR}_t + \beta_3 \log \text{GDPG}_t + \beta_4 \log \text{SSECI}_t \\ + \beta_5 \text{LM}_t + \beta_6 \lambda_{t-2\Delta t} + \beta_7 \lambda_{t-\Delta t} + \epsilon_t.$$

are reported for all the seven ratings (AAA, AA+, AA, AA-, A+, A, BBB) as well as the average. The sample first order autocorrelation of the residuals  $\varrho$  is also reported in the last column. In the table, \*\*\*, \*\*, and \* entries represent significance at the 1%, 5%, and 10% levels, respectively. The significance levels are calculated based on the Bootstrapped standard errors detailed in Appendix A.

	Intrcpt	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\beta_7$	$\varrho$
AAA	0.09	<b>-0.84</b> ***	-0.00	0.74	0.11	-0.03	0.00	<b>0.95</b> ***	-0.01
AA+	-0.23	<b>-0.66</b> ***	-0.01	0.52	0.41	-0.00	-0.05	<b>1.03</b> ***	-0.01
AA	-0.49	<b>-0.46</b> *	-0.01	0.18	0.77	0.05	<b>-0.22</b> ***	<b>1.21</b> ***	0.02
AA-	-0.47	<b>-0.60</b> **	-0.01	-0.14	0.83	0.08	-0.11	<b>1.10</b> ***	0.00
A+	0.88	<b>-0.77</b> ***	<b>-0.05</b> ***	-0.63	-0.64	-0.01	-0.07	<b>1.06</b> ***	-0.01
A	<b>2.15</b> *	-0.33	<b>-0.20</b> ***	<b>-2.53</b> ***	-1.56	0.07	-0.06	<b>1.06</b> ***	-0.00
BBB	1.99	-0.41	<b>-0.22</b> ***	<b>-3.02</b> ***	-1.13	0.20	-0.03	<b>1.03</b> ***	-0.02
average	0.16	<b>-0.48</b> **	-0.01	0.10	0.02	-0.04	<b>-0.19</b> ***	<b>1.18</b> ***	-0.00

**Figure 8:** Average  $\lambda$  vs credit risk measures and various control variables

In this figure, the dynamic of average  $\lambda$  across all the ratings from Apr 2006 to Mar 2013 is plotted against “log of CVI (log CVI)”, “one year forward average default rate (1yr Fwd ave DR)”, “non-performing bank loan rate (NPLR)”, “log of GDP Growth (log GDPG)”, “log of SSECI (log SSECI)”, and “liquidity measure (LM)”, and in (a), (b), (c), (d), (e), and (f).



### 5.3 Countercyclical risk premia

As shown in Section 4.4,  $\psi^Q - \psi^P$  is a model implied measure of short term risk premia associated with corporate-specific risks. Since not all the MPR parameters of  $\lambda_1$  and  $\lambda_2$  are significant, we use  $(\hat{\eta}_i^j)^*$  instead of its point estimate  $\hat{\eta}_i^j$  when constructing  $\psi^P$ , where  $i = 1, 2, j = 0, 1$ ,

$$(\hat{\eta}_i^j)^* = \begin{cases} \hat{\eta}_i^j & \text{if } |\hat{\eta}_i^j| > \text{s.e.}\hat{\eta}_i^j \\ 0 & \text{if } |\hat{\eta}_i^j| \leq \text{s.e.}\hat{\eta}_i^j \end{cases}.$$

$\hat{\eta}_i^j$  and  $\text{s.e.}\hat{\eta}_i^j$  are shown in Table 5. The dynamics of  $\psi^Q - \psi^P$ ,  $\psi^Q$ , and  $\psi^P$  of different ratings are plotted in Figure 9. The risk premia for the rating AAA are in average lower than others, this might indicate that the bonds of AAA rating have better liquidity, so investors are willing to pay lower premia. Interestingly, the risk premia for the ratings A and BBB are least volatile, and those of BBB are even constant over the whole sample period. The constant risk premium for BBB rated bonds is merely a result of insignificant estimates of  $\eta_1^1$ ,  $\eta_2^0$ , and  $\eta_2^1$ . In other words, the time-varying property of BBB's risk premium is statistically insignificant. This might be because bonds with lowest ratings are least traded.

Following the same procedure in Section 5.2, we inspect the relationship between the risk premia and aggregate credit risk factors and other control variables. Specifically, we run the following regression for the test,

$$\begin{aligned} \psi_t^Q - \psi_t^P = & \text{Intercept} + \beta_1 \text{adj-log CVI}_t + \beta_2 \text{adj-ADR}_t + \beta_3 \text{NPLR}_t + \beta_4 \log \text{GDPG}_t \\ & + \beta_5 \log \text{SSECI}_t + \beta_6 \text{LM}_t + \beta_7 (\psi^Q - \psi^P)_{t-2\Delta t} + \beta_8 (\psi^Q - \psi^P)_{t-\Delta t} + \epsilon_t. \end{aligned}$$

Table 10 presents the regression results. As shown, we do not observe significantly relationships between the risk premium and adj-log CVI and adj-ADR. In contrast, we find significant correlations between the risk premium and NPLR, log GDPG and log SSECI. For all the ratings lower than AAA, the coefficients of log GDPG are significantly negative. This finding is consistent with the evidence in Adrian et al. (2010) about the relationship between the GDP growth and the macro risk premium. The intuition behind this negative relationship is that when the economic growth is faster (slower), there is more (less) capital

**Table 10:** Results from the regression of  $\psi^Q - \psi^P$  on adj-log CVI, adj-ADR and other control variables

In this table the results from the following regression

$$\begin{aligned} \psi_t^Q - \psi_t^P = & \text{Intercept} + \beta_1 \text{adj-log CVI}_t + \beta_2 \text{adj-ADR}_t + \beta_3 \text{NPLR}_t + \beta_4 \log \text{GDPG}_t \\ & + \beta_5 \log \text{SSECI}_t + \beta_6 \text{LM}_t + \beta_7 (\psi^Q - \psi^P)_{t-2\Delta t} + \beta_8 (\psi^Q - \psi^P)_{t-\Delta t} + \epsilon_t, \end{aligned}$$

are reported for the six ratings, AAA, AA+, AA, AA-, A+, and A as well as the average (BBB is not reported because the model implied risk premium for BBB is constant all time). The sample first order autocorrelation of the residuals  $\varrho$  is also reported in the last column. In the table, \*\*\*, \*\*, and \* entries represent significance at the 1%, 5%, and 10% levels, respectively. The significance levels are calculated based on the Bootstrapped standard errors detailed in Appendix A.

	Intrcpt	$\beta_1$	$\beta_2$	$\beta_3$	$\beta_4$	$\beta_5$	$\beta_6$	$\beta_7$	$\beta_8$	$\varrho$
AAA	-0.15	0.46	-0.12*	-0.00	-0.21	0.25**	-0.01	-0.46***	1.41***	0.03
AA+	0.47	-3.87	-0.12	-0.00	-0.39*	0.36*	-0.04	-0.48***	1.39***	0.02
AA	-1.20	5.54	-0.38	-0.02*	-1.97**	1.32*	-0.08	-0.50***	1.42***	-0.01
AA-	-0.27	-1.68	-0.23	-0.02**	-0.65*	0.89**	0.00	-0.55***	1.49***	0.00
A+	0.40	-3.12	-0.08	-0.03**	-1.56**	0.73	-0.03	-0.60***	1.50***	0.03
A	0.65**	-1.82	-0.06	-0.02**	-0.48***	0.04	-0.01	-0.60***	1.52***	0.03
average	1.12*	-4.72	-0.04	-0.01*	-1.20**	-0.09	-0.37***	-0.28***	1.20***	-0.01

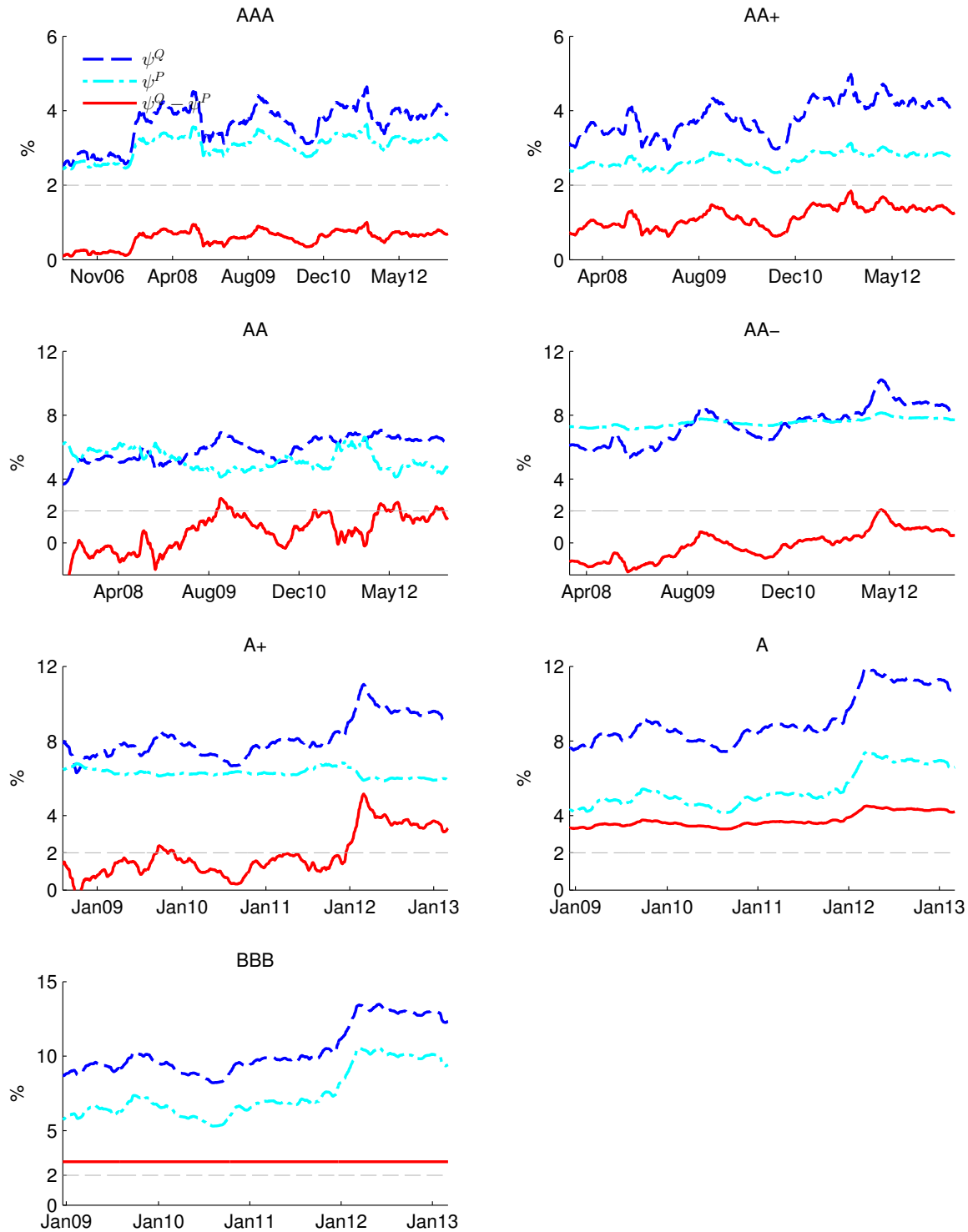
for investment and, therefore, lower (higher) risk premia are requested for bearing risks. This observation somewhat validates our risk premium calculation.

For higher ratings, AAA, AA+, AA, and AA-, the risk premium is positively and significantly related to log SSECI. Given the fact that the equity market in China is much stronger than the corporate bond market (Allen et al., 2009), this positive correlation confirms that for the corporate bond investors the equity market is a comparable substitute to the corporate bond market. Therefore it is “costly” to invest in the corporate bond market. When the equity market performs better (worse), the opportunity cost for corporate bond investors becomes higher (lower), therefore they request higher (lower) risk premia.

For all the ratings lower than AA+, the coefficients of NPLR are significantly negative. As mentioned earlier, the bank loan market in China is much larger than the corporate

**Figure 9:** Time varying risk premia

This figure plots the dynamic of the risk premium measure  $\psi^Q - \psi^P$  along with the central tendency variables  $\psi^Q$  (under the measure Q) and  $\psi^P$  (under the measure P) for all the seven ratings (AAA, AA+, AA, AA-, A+, A, and BBB). A gray dash line at 2% is also plotted to facilitate a rough comparison of the risk premia across the ratings.

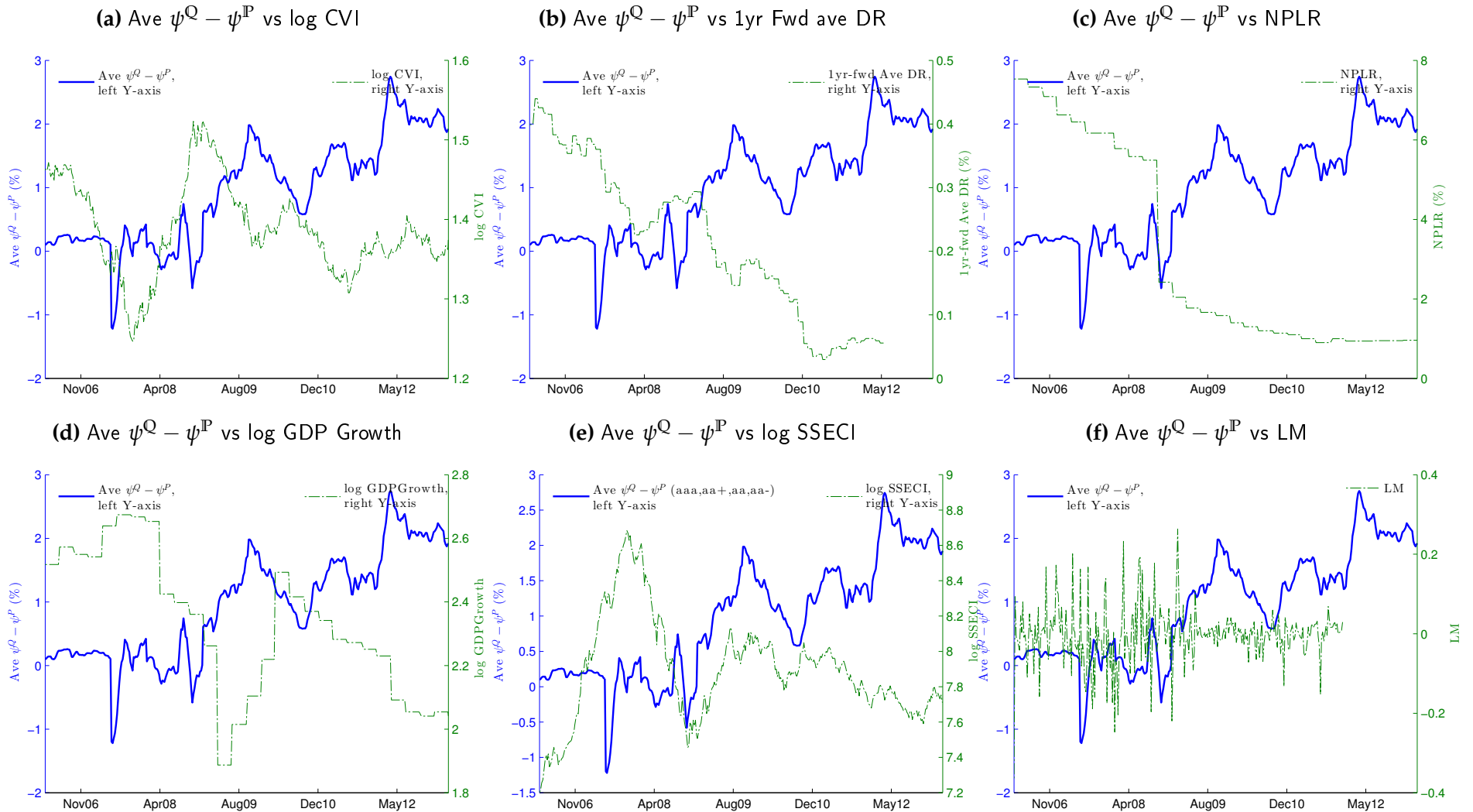


bond market. This negative relationship between risk premium and NPLR indicates that the opportunity cost argument mentioned above also applies between bank loan market and corporate bond market. This is especially sensible since, as mentioned in Section 3.2, the bank loan data includes a lot of smaller firms that have only bank loans and do not issue corporate bonds.

It is also worth noting that, for the average rating, we observe significant and negative coefficient for the LM. This means that, in general, when the liquidity is lower (higher), corporate bond investors request higher (lower) risk premia. Although we do not have this result for  $\lambda$ , we do observe significant correlation between the LM and the YS (see Tables 2 and 3). This indicates that the LM is negatively related to the risk premium embedded in the YS. This result is consistent with our previous conjecture that the illiquid secondary market prevents the bond prices from revealing the true underlying risks. When the secondary market is noticeably illiquid, the risk premium in the YS will be significantly and negatively affected by the liquidity of the market, which is positively related to the aggregate credit risk. Therefore, the bond prices would hardly reveal the true underlying credit risks in the credit bond market.

**Figure 10:** Average  $\psi^Q - \psi^P$  vs credit risk measures and various control variables

In this figure, the dynamic of average  $\psi^Q - \psi^P$  across all the ratings from Apr 2006 to Mar 2013 is plotted against “log of CVI (log CVI)”, “one year forward average default rate (1yr Fwd ave DR)”, “non-performing bank loan rate (NPLR)”, “log of GDP Growth (log GDPG)”, “log of SSECI (log SSECI)”, and “liquidity measure (LM)” in (a), (b), (c), (d), (e), and (f).





## 5.4 An explanation for the counter-credit-risk puzzle

Taken together, the empirical results indicate that the corporate specific risk factor is negatively and strongly related to the aggregate credit risk measures, and its risk premia are countercyclical and positively related to the stock index (for higher rating bonds) and negatively related to the non-performing bank loan rate (for lower rating bonds). The counter-credit-risk corporate bond factor presents an interesting puzzle which completely violates the intended function of the corporate bond market. The puzzle per se deserves great attention and deeper investigation. Here we offer an intuitive explanation for this puzzle based on the current development of China's corporate bond market and leave further exploring to future research.

**A buy-side driven market:** Given the relatively stable growth of corporate bond issuances, the increase of the corporate bond supplies, i.e., the total number of bonds available for trading, is relatively stable over time. Therefore the dynamic of the corporate-specific factor in the corporate yield spreads is mainly driven by the buy-side force, i.e., by the money flowing into and out of the market.

**A corporate bond market with “zero tolerance” for default for two decades:** As mentioned earlier, most of the corporate bonds are issued by state-owned and state-run companies, or large/mid-cap Chinese companies. Although after Oct 2007 Chinese corporate bonds were no longer guaranteed by state-owned banks,<sup>14</sup> they are still explicitly or implicitly backed by their high-profile parent companies or local governments. As of the end of Feb 2014, there had been NOT a single default occurring in China's corporate bond market. Zero credit risk has almost become a conventional assumption among investors in the market.

**Attracting credit risk sensitive capital from other markets:** Unlike the credit bond market, the credit risk is one of the main concerns in the equity and the bank loan markets. Institutions, which have access to both the corporate bond and the equity markets (such as investment banks, mutual funds, and hedge funds), or both the corporate bond and the bank loan markets (such as commercial banks), will shift the capital

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<sup>14</sup> See “Opinions of China Banking Regulatory Commission on Effectively Preventing the Security Risks of Corporate Bonds” issued by China Banking Regulatory Commission on 10 Oct 2007.

in their investment portfolio from high credit risk exposure positions to the corporate bonds when the credit condition deteriorates. The positive relationship between LM and the aggregate credit risk mentioned before also supports this story: when credit risk in the equity and bank loan markets increases, investors flock into credit bonds, causing their liquidity to increase. This interesting conjecture apparently deserves further investigation, as it might well serve as an empirical Flight-to-Safety episode sought by Inghelbrecht et al. (2013). A formal way to test this conjecture is to look at the flow of funds between the credit bond market and other markets. This exercise, however, calls for much more data and analysis that are way beyond the scope of the current paper. We therefore leave this interesting study to future research.

**Counter-credit-risk corporate yields:** As analyzed above, when the credit risk is high, the money from credit risk sensitive market flows into the corporate bond market which is believed to be credit-risk-free, thus reduces the corporate yields. In contrast, when the credit condition ameliorates, the opportunity cost of investing in the corporate bond market increases (the substitution effect reflected in the risk premia mentioned in Section 5.3). Therefore the money flows out of the corporate bond market and the demand for the corporate bonds decreases. This increases the corporate yields.

**Inefficient capital allocation, moral hazard, and credit risk accumulation:** One of the most obvious consequences of the resulting counter-credit-risk corporate yield spreads is an inefficient capital allocation effect. The distorted function of the corporate bond market completely mis-prices the credit risk in the opposite direction. The capital is directed to the market not because of its efficiency in deploying the capital but of a free insurance against the credit risk. Another consequence that should not be neglected is the moral hazard issue. In this distorted market corporate bond issuers are able to raise money more easily when the credit condition is bad than when it is good. This inevitably gives bond issuers incentives to take more risks when proposing projects. The joint effect of the inefficient capital allocation and the moral hazard is an accumulation of credit risk. Given the fast growth of China's corporate bond market, the accumulating credit risk would become rather harmful to China's finan-

cial system. A growing and malfunctioning corporate bond market (unintentionally) comes with a brewing crisis.

## 6 Conclusions

Employing risk-free and corporate zero yield data with aggregate credit risk measures and macroeconomic variables, we document a puzzle of counter-credit-risk corporate yield spreads in China's credit bond market. Specifically, we find a significant and negative relationship between the corporate yield spreads and aggregate credit risk measures, and the result is robust to the choice of the aggregate credit risk measures and to the inclusion of various control variables (business cycle, equity market performance, aggregate liquidity of bond markets). This puzzle per se deserves great attention and deeper investigation, since a corporate bond market is naturally expected to discover the price of credit risks in a directly proportional way.

We also find that the corporate yield spreads are positively related to the slope (for the CB market) and the curvature (for both the CB and CPs&MTNs markets). This finding contradicts theoretical predictions (Longstaff and Schwartz, 1995; Collin-Dufresne et al., 2001), and is possibly attributed to the counter-credit-risk puzzle. Another finding is the positive (negative) relationship between the risk premia and the stock index (the non-performing bank loan rate), which indicates that for the credit bond investors the equity market (the bank loan) is a comparable substitute to the credit bond market, and helps explain the puzzle from an opportunity cost perspective.

We interpret the puzzle as a symptom of the immaturity of China's credit bond market in two specific aspects: an illiquid secondary market and a distorted credit risk pricing mechanism. The emergence of the puzzle (symptom) urges policy makers to focus more on the development of these two aspects.

# Bibliography

- Adrian, T., E. Moench, and H. S. Shin (2010). Macro risk premium and intermediary balance sheet quantities. *IMF Economic Review* 58(1), 179–207. 3, 36
- Allen, F., J. Qian, M. Qian, and M. Zhao (2009). A review of china's financial system and initiatives for the future. In J. R. Barth, J. A. Tatom, and G. Yago (Eds.), *China's Emerging Financial Markets*, Volume 8 of *The Milken Institute Series on Financial Innovation and Economic Growth*, pp. 3–72. Springer US. 37
- Altman, E. I., J. B. Caouette, and P. Narayanan (1998). Credit-risk measurement and management: The ironic challenge in the next decade. *Financial Analysts Journal* 54(1), 7–11. 11
- Ang, A. and F. A. Longstaff (2013). Systemic sovereign credit risk: Lessons from the us and europe. *Journal of Monetary Economics* 60(5), 493–510. 16
- Bai, J., P. Collin-Dufresne, R. S. Goldstein, and J. Helwege (2012). On bounding credit event risk premia. Technical report, Federal Reserve Bank of New York. 24
- Bao, J. (2009). Structural models of default and the cross section of corporate bond yield spreads. Technical report, Working Paper, MIT. 31
- Chen, A. H., S. C. Mazumdar, and R. Surana (2011). China's corporate bond market development. *Chinese Economy* 44(5), 6–33. 2
- Cheridito, P., D. Filipović, and R. Kimmel (2007). Market price of risk specifications for affine models: Theory and evidence. *Journal of Financial Economics* 83(1), 123–170. 22
- Collin-Dufresne, P., R. S. Goldstein, and J. S. Martin (2001). The determinants of credit spread changes. *The Journal of Finance* 56(6), 2177–2207. 3, 26, 31, 43
- Dai, Q. and K. Singleton (2000). Specification analysis of affine term structure models. *Journal of Finance* 55(5), 1943–1978. 50, 51
- Dai, Q. and K. Singleton (2002). Expectation puzzles, time-varying risk premia, and affine models of the term structure. *Journal of Financial Economics* 63(3), 415–441. 16

- Driessen, J. (2005). Is default event risk priced in corporate bonds? *Review of Financial Studies* 18(1), 165–182. 24
- Duan, J. and J. Simonato (1999). Estimating and Testing Exponential-Affine Term Structure Models by Kalman Filter. *Review of Quantitative Finance and Accounting* 13(2), 111–135. 25
- Duan, J.-C., J. Sun, and T. Wang (2012). Multiperiod corporate default prediction – a forward intensity approach. *Journal of Econometrics*. 11
- Duffee, G. (1999). Estimating the price of default risk. *Review of Financial Studies* 12(1), 197–226. 16, 25
- Duffee, G. (2002). Term premia and interest rate forecasts in affine models. *The Journal of Finance* 57(1), 405–443. 22
- Duffee, G. R. (1998). The relation between treasury yields and corporate bond yield spreads. *The Journal of Finance* 53(6), 2225–2241. 26
- Duffie, D. and R. Kan (1996). A yield-factor model of interest rates. *Mathematical Finance* 6(4), 379–406. 49
- Duffie, D., J. Pan, and K. Singleton (2000). Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 1343–1376. 19
- Duffie, D., L. Saita, and K. Wang (2007). Multi-period corporate default prediction with stochastic covariates. *Journal of Financial Economics* 83(3), 635–665. 11
- Duffie, D. and K. Singleton (1997). An econometric model of the term structure of interest-rate swap yields. *Journal of finance*, 1287–1321. 18
- Duffie, D. and K. Singleton (1999). Modeling term structures of defaultable bonds. *Review of Financial Studies* 12(4), 687–720. 18
- Elton, E. J., M. J. Gruber, D. Agrawal, and C. Mann (2001). Explaining the rate spread on corporate bonds. *The Journal of Finance* 56(1), 247–277. 31

- Eom, Y. H., J. Helwege, and J.-z. Huang (2004). Structural models of corporate bond pricing: An empirical analysis. *Review of Financial studies* 17(2), 499–544. 31
- Filipović, D. and A. B. Trolle (2013). The term structure of interbank risk. *Journal of Financial Economics* 109(3), 707–733. 16
- Giese, J. (2008). Level, slope, curvature: characterising the yield curve in a cointegrated var model. *Economics: The Open-Access, Open-Assessment E-Journal* 2. 26
- Granger, C. W. and P. Newbold (1974). Spurious regressions in econometrics. *Journal of econometrics* 2(2), 111–120. 13
- Hale, G. (2007). Prospects for china’s corporate bond market. *FRBSF Economic Letter* (Mar 16). 2
- Han, B. (2008). Investor sentiment and option prices. *Review of Financial Studies* 21(1), 387–414. 13
- Huang, H. and N. Zhu (2009). The chinese bond market: Historical lessons, present challenges, and future perspectives. In *China’s Emerging Financial Markets*, pp. 523–546. Springer. 2
- Huang, J.-Z. and M. Huang (2012). How much of the corporate-treasury yield spread is due to credit risk? *Review of Asset Pricing Studies* 2(2), 153–202. 31
- Huang, J.-Z. and W. Kong (2003). Explaining credit spread changes: New evidence from option-adjusted bond indexes. *The Journal of Derivatives* 11(1), 30–44. 12
- Inghelbrecht, K., G. Bekaert, L. Baele, and M. Wei (2013). Flights to safety. *NBER WORKING PAPER SERIES* 19095, 1–51. 42
- Jappelli, T., M. Pagano, and M. Bianco (2005). Courts and banks: Effects of judicial enforcement on credit markets. *Journal of Money, Credit, and Banking* 37(2), 223–244. 11
- Jarrow, R., D. Lando, and F. Yu (2005). Default risk and diversification: Theory and empirical implications. *Mathematical Finance* 15(1), 1–26. 24

- Jegadeesh, N. and G. G. Pennacchi (1996). The behavior of interest rates implied by the term structure of eurodollar futures. *Journal of Money, Credit and Banking* 28(3), 426–446. 22
- Joslin, S., K. Singleton, and H. Zhu (2011). A new perspective on gaussian dynamic term structure models. *Review of Financial Studies*. 16, 25, 50
- Keele, L. and N. J. Kelly (2006). Dynamic models for dynamic theories: The ins and outs of lagged dependent variables. *Political Analysis* 14(2), 186–205. 13
- Litterman, R. and J. Scheinkman (1991). Common factors affecting bond returns. *The Journal of Fixed Income* 1(1), 54–61. 26
- Longstaff, F., S. Mithal, and E. Neis (2005). Corporate yield spreads: Default risk or liquidity? New evidence from the credit default swap market. *Journal of Finance* 55(5), 2213–2253. 16, 24
- Longstaff, F. A., J. Pan, L. H. Pedersen, and K. J. Singleton (2011). How sovereign is sovereign credit risk? *American Economic Journal: Macroeconomics* 3(2), 75–103. 24
- Longstaff, F. A. and E. S. Schwartz (1995). A simple approach to valuing risky fixed and floating rate debt. *The Journal of Finance* 50(3), 789–819. 3, 26, 43
- Ma, G. (2006). Sharing china’s bank restructuring bill. *China & World Economy* 14(3), 19–37. 2
- Martinez Peria, M. S. and A. Mody (2004). How foreign participation and market concentration impact bank spreads: Evidence from latin america. *Journal of Money, Credit, and Banking* 36(3), 511–537. 11
- Mu, H. (2006). The development of china’s bond market. *BIS Paper 26 Developing Corporate Bond Markets in Asia*, 56–60. 2
- Pan, J. and K. Singleton (2008). Default and recovery implicit in the term structure of sovereign CDS spreads. *The Journal of Finance* 63(5), 2345–2384. 24

- Pastor, L. and R. F. Stambaugh (2003). Liquidity risk and expected stock returns. *Journal of political economy* 111(3), 642–685. 12
- RMI-Staff (2012, 07). Cvi white paper. Technical report, Risk Management Institute, National University of Singapore. 11
- RMI-Staff (2014). Nus-rmi credit rating initiative technical report. Technical report, Risk Management Institute, National University of Singapore. 9, 10
- Shen, Y., M. Shen, Z. Xu, and Y. Bai (2009). Bank size and small-and medium-sized enterprise (sme) lending: Evidence from china. *World Development* 37(4), 800–811. 11
- Shiller, R. J. (1979). The volatility of long-term interest rates and expectations models of the term structure. *The Journal of Political Economy*, 1190–1219. 24
- Suzuki, Y., M. Miah, J. Yuan, et al. (2008). China's non-performing bank loan crisis: the role of economic rents. *Asian-Pacific Economic Literature* 22(1), 57–70. 2
- Wildau, G. (2012). Rpt-update 1-chinese fibre maker to repay bond, skirting country's first default. online article, Reuters.com. 7
- Zheng, T. and X. Wang (2010). Real time estimates of chinese output gap and reliability analysis [j]. *Economic Research Journal* 10, 011. 1, 12
- Zhou, X. (2003). Explore market position to promote corporate bond development. In *International Seminar on Bond Market Development: Opportunities and Challenges*. 2
- Zhou, X. (2006). China's corporate bond market development: lessons learned. *BIS Paper* 26 *Developing Corporate Bond Markets in Asia*, 7–10. 2



# Appendices

## A Bootstrapped standard errors

In this appendix, we outline the details of the bootstrapped standard errors. The calculation consists of the following steps:

1. Estimate the coefficients using OLS, and save the estimates and residuals;
2. Estimate an AR(1) model for the residuals;
3. Simulate residuals based on the estimated AR(1) from step 2, and combine with the estimates saved in step 1 to generate the LHS variable;
4. Based the generated LHS variable from step 3 and the original RHS variables,<sup>15</sup> estimate a new set of coefficients;
5. Repeat steps 3 to 4 one thousand times, the bootstrapped standard errors are the standard deviations of the one thousand sets of coefficient estimates.

## B Risk-free bond model derivation

This appendix derives the risk-free zero coupon bond model. The short rate  $r_t$  is a affine function of the state variables  $Z_t$ ,

$$r_t = \varphi + \mathbf{C}Z_t.$$

Since  $Z_t$  is a multivariate Gaussian process:

$$dZ_t = \mathbf{A}Z_t dt + \mathbf{B}dW_t,$$

by Duffie and Kan (1996), the risk-free zero coupon bond  $P(t, t+x)$  is given by

$$P(t, t+x) = \exp(\mathbf{H}(x) - \mathbf{F}(x)^\top Z_t)$$

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<sup>15</sup> Here the original RHS variables exclude the lagged the LHS variable which is generated from step 3.

where  $\mathbf{H}(x)$  and  $\mathbf{F}(x)$  satisfy the ordinary differential equations:

$$\frac{d\mathbf{H}(x)}{dx} = \frac{1}{2}\mathbf{F}(x)^\top \mathbf{B}\mathbf{B}^\top \mathbf{F}(x) - \varphi \text{ and } \frac{d\mathbf{F}(x)}{dx} = \mathbf{A}^\top \mathbf{F}(x) + \mathbf{C}^\top$$

with the boundary conditions  $\mathbf{H}(0) = 0, \mathbf{F}(0) = \mathbf{0}_{3 \times 1}$ .  $\mathbf{F}(x)$  is easily solved and given by

$$\mathbf{F}(x)^\top = \mathbf{C} \int_0^x \exp(\mathbf{A}s) ds = (\mathbf{C}(x) - \mathbf{C}) \mathbf{A}^{-1}.$$

Then it follows

$$\begin{aligned} \mathbf{H}(x) &= \frac{1}{2} \int_0^x (\mathbf{C}(s) - \mathbf{C}) \mathbf{A}^{-1} \mathbf{B}\mathbf{B}^\top (\mathbf{A}^\top)^{-1} (\mathbf{C}(s) - \mathbf{C})^\top ds - \varphi x \\ &= - \int_0^x \mathbf{C}(s) \left( \mathbf{A}^{-1} \mathbf{B}\mathbf{B}^\top (\mathbf{A}^\top)^{-1} \right) \left( \mathbf{C}^\top - \frac{\mathbf{C}(s)^\top}{2} \right) ds - \left( \varphi - \frac{1}{2} \mathbf{C} \left( \mathbf{A}^{-1} \mathbf{B}\mathbf{B}^\top (\mathbf{A}^\top)^{-1} \right) \mathbf{C}^\top \right) x. \end{aligned}$$

Therefore the zero yield  $y_t^x$  is given by

$$y_t^x = -\frac{\log(P(t, t+x))}{x} = -\frac{\mathbf{H}(x)}{x} + \frac{\mathbf{F}(x)^\top}{x} Z_t.$$

In fact, the model is a variant of  $\mathbb{A}_0(3)$  developed in Dai and Singleton (2000). If we take  $\mathbf{B}^{-1}Z_t$  as the state variable, the model will coincide with the canonical representation of  $\mathbb{A}_0(3)$  proposed in Dai and Singleton (2000). The reason we use the above representation is to be consistent with Joslin et al. (2011), so that we would be able to apply their estimation methodology.

In order to apply Joslin et al. (2011) approach, the model is rotated such that the state variable is  $PC_t$  (the first three principal components of risk-free zero yields): by definition of the principal components, we have

$$PC_t = wm \cdot A_m Z_t + wm \cdot C_m$$

where  $wm$  is the weight matrix for PCs, i.e.,  $PC_t = wm \cdot y_t, y_t = \begin{bmatrix} y_t^{x_1} & y_t^{x_2} & \dots & y_t^{x_n} \end{bmatrix}^\top$  is

the zero yield curve at time  $t$ ;

$$\begin{aligned} A_m &= \begin{bmatrix} \frac{\mathbf{F}(x_1)}{x_1} & \frac{\mathbf{F}(x_2)}{x_2} & \dots & \frac{\mathbf{F}(x_n)}{x_n} \end{bmatrix}^\top \\ C_m &= - \begin{bmatrix} \frac{\mathbf{H}(x_1)}{x_1} & \frac{\mathbf{H}(x_2)}{x_2} & \dots & \frac{\mathbf{H}(x_n)}{x_n} \end{bmatrix}^\top. \end{aligned}$$

Therefore by Dai and Singleton (2000) we have,

$$r_t = \varphi^{PC} + \rho PC_t$$

where  $\varphi^{PC} = \varphi - \mathbf{C} (wm \cdot A_m)^{-1} wm \cdot C_m$ ,  $\rho = \mathbf{C} (wm \cdot A_m)^{-1}$ ;

$$dPC_t = \mathbf{A}^{PC,Q} \left( PC_t - \mu^{PC,Q} \right) dt + \mathbf{B}^{PC} dW_t$$

where  $\mathbf{A}^{PC,Q} = (wm \cdot A_m) \mathbf{A} (wm \cdot A_m)^{-1}$ ,  $\mu^{PC,Q} = wm \cdot C_m$ , and  $\mathbf{B}^{PC} = wm \cdot A_m \mathbf{B}$ .